

$$\begin{Bmatrix} \mathbf{P}^* \\ \mathbf{P}^0 \end{Bmatrix} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix} \begin{Bmatrix} \mathbf{D}^* \\ \mathbf{D}^0 \end{Bmatrix}$$

$$\mathbf{P}^{**} = \mathbf{K}^* \mathbf{D}^*,$$

$$\mathbf{P}^{**} = \mathbf{P}^* - \mathbf{K}_{12} \mathbf{K}_{22}^{-1} \mathbf{P}^0$$

$$\mathbf{K}^* = \mathbf{K}_{11} - \mathbf{K}_{12} \mathbf{K}_{22}^{-1} \mathbf{K}_{21}$$

$$\mathbf{M}\ddot{\mathbf{D}} + \mathbf{K}\mathbf{D} = \mathbf{P}(t)$$

$$(\mathbf{K} - \lambda \mathbf{M})\boldsymbol{\phi} = \mathbf{0}, \quad \lambda = \omega^2$$

$$\det(\mathbf{K} - \lambda \mathbf{M}) = 0$$

$$\boldsymbol{\phi}_i^T \mathbf{M} \boldsymbol{\phi}_j = \delta_{ij}$$

$$\boldsymbol{\phi}_i^T \mathbf{K} \boldsymbol{\phi}_j = \lambda_i \delta_{ij}$$

$$\boldsymbol{\Phi} = [\boldsymbol{\phi}_1 \quad \boldsymbol{\phi}_2 \quad \boldsymbol{\phi}_3 \quad \dots \quad \boldsymbol{\phi}_n]$$

$$\boldsymbol{\Lambda} = \text{diag}(\lambda_1 \quad \lambda_2 \quad \lambda_3 \quad \dots \quad \lambda_n)$$

$$\mathbf{q} = \boldsymbol{\Phi} \boldsymbol{\eta}$$

$$\mathcal{M} = \boldsymbol{\Phi}^T \mathbf{M} \boldsymbol{\Phi}$$

$$\mathcal{K} = \boldsymbol{\Phi}^T \mathbf{K} \boldsymbol{\Phi}$$

$$\mathcal{F} = \boldsymbol{\Phi}^T \mathbf{P}(t)$$

Bars:

$$\mathbf{m} = \frac{\rho AL}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}, \quad \mathbf{m}_L = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \mathbf{k} = \frac{EA}{L} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad l = \frac{x_2 - x_1}{L} = \cos \alpha$$

$$m = \frac{y_2 - y_1}{L} = \sin \alpha$$

$$\mathbf{m} = \frac{\rho AL}{6} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix}, \quad \mathbf{m}_L = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad \mathbf{k} = \frac{EA}{L} \begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

Beams:

$$\underline{\mathbf{m}} = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L^2 & -22L & 4L^2 \end{bmatrix}, \quad \mathbf{m}_L = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \underline{\mathbf{k}} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

$$\underline{\mathbf{m}} = \frac{\rho AL}{420} \begin{bmatrix} 156 & 22L & 54 & -13L & 0 \\ 22L & 4L^2 & 13L & -3L^2 & 0 \\ 54 & 13L & 156 & -22L & 0 \\ -13L & -3L^2 & -22L & 4L^2 & 0 \\ 0 & 0 & 0 & 0 & 420 \end{bmatrix}, \quad \mathbf{m}_L = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}, \quad \underline{\mathbf{k}} = \frac{EI}{L^3} \begin{bmatrix} 12 & 6L & -12 & 6L & 0 \\ 6L & 4L^2 & -6L & 2L^2 & 0 \\ -12 & -6L & 12 & -6L & 0 \\ 6L & 2L^2 & -6L & 4L^2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$