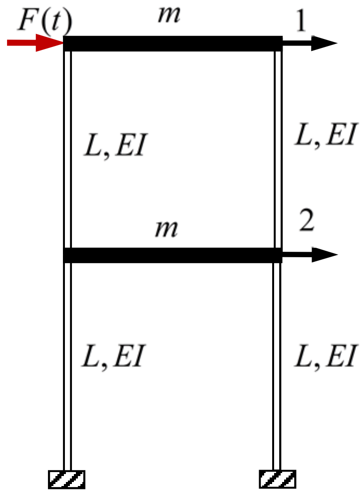
RAK-33046 Dynamics of Structures

Exercise 9 1.4.2016



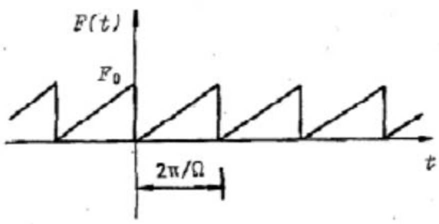
$$\boldsymbol{K} = \frac{EI}{L^3}\begin{bmatrix} 24 & -24 \\ -24 & 48 \end{bmatrix}$$

$$\boldsymbol{M} = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$$

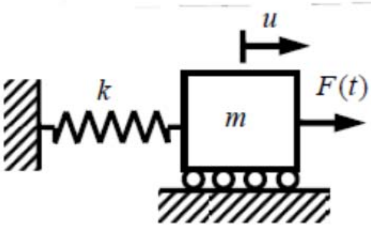
1. Horizontal beams are rigid with the mass *m*, and vertical beams have no mass and no axial flexibility.

   a) The force $F(t) = F_0 \sin \Omega t$ is acting on the first DOF, where $\Omega = 1,5\omega_1$ and ω_1 is the lowest natural eigenfrequency. Determinate the steady state response when the damping ratio is $\zeta = 0,10$ (for both eigenmodes)

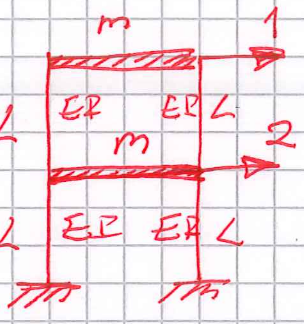
   b) If $\Omega = (\omega_1 + \omega_2)/2$, where ω_2 is the second natural eigenfrequency, determinate the steady state response for the undamped system.

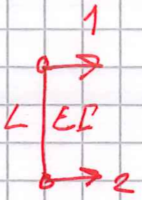


2. Esitä kuvan heräte *FOURIER*-sarjana ja määritä vaimentamattoman yhden vapausasteen systeemin vaste $u(t)$. Piirrä herätteen ja vasteen kuvaajia, jotka vastaavat *FOURIER*-sarjan alkupään termejä. $\Omega = 2$ 1/s, $\Omega/\omega = 0,80$



Express the harmonic excitation by Fourier series and determinate the response of the undamped 1DOF vibrator. Draw curves for the excitation approximation and the response using one and three terms of Fourier series.
$\Omega = 2$ rad/sec, $\Omega/\omega = 0,80$





$$k^{(e)} = \frac{EI}{L^3}\begin{bmatrix} 12 & -12 \\ -12 & 12 \end{bmatrix}$$

$$K = \frac{EI}{L^3}\begin{bmatrix} 24 & -24 \\ -24 & 48 \end{bmatrix}$$

$$= \frac{EI}{L^3}\begin{bmatrix} 24 & -24 \\ -24 & 48 \end{bmatrix}$$

$$M = m\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M\ddot{q} + Kq = \begin{pmatrix} F_0 \sin \Omega t \\ 0 \end{pmatrix} \qquad \lambda = \bar{\lambda}\,\frac{EI}{mL^3}$$

$$\cancel{(M - \lambda K)\phi = 0}$$

$$(K - \bar{\lambda}M)\underline{\phi} = \underline{0} \qquad \det\left(\begin{bmatrix} 24-\bar{\lambda} & -24 \\ -24 & 48-\bar{\lambda} \end{bmatrix}\right) = 0$$

$$(24-\bar{\lambda})(48-\bar{\lambda}) - 24^2 = 0$$

$$\bar{\lambda}^2 - 72\bar{\lambda} + 576 \qquad \lambda_{1,2} = \frac{72 \pm \sqrt{72^2 - 4 \cdot 576}}{2}$$

$$\lambda_1 = 9{,}16718 \qquad \omega_1 = 3{,}02773\sqrt{}$$

$$\underline{\phi}_1 = \begin{pmatrix} 1 \\ k \end{pmatrix} \quad \begin{pmatrix} 1 \\ 0{,}61803 \end{pmatrix} \qquad \lambda_2 = 62{,}8328 \qquad \omega_2 = 7{,}92671\sqrt{}$$

$$\underline{\phi}_2 = \begin{pmatrix} 1 \\ -1{,}61803 \end{pmatrix} \qquad \underline{q} = \underline{\Phi}\,\underline{\eta}$$

$$\underline{\Phi} = \begin{bmatrix} 1 & 1 \\ 0{,}61803 & -1{,}61803 \end{bmatrix} \qquad \underline{\Phi}^T M \underline{\Phi}\,\ddot{\underline{\eta}} + \underline{\Phi}^T K \underline{\Phi}\,\underline{\eta} = \underline{\Phi}^T \begin{pmatrix} F_0 \sin \\ 0 \end{pmatrix}$$

$$m\begin{bmatrix} 1{,}3826 & 0 \\ 0 & 3{,}6180 \end{bmatrix}\begin{pmatrix} \ddot{\eta}_1 \\ \ddot{\eta}_2 \end{pmatrix} + \frac{EI}{L^3}\begin{bmatrix} 12{,}6687 & 0 \\ 0 & 227{,}3313 \end{bmatrix}\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} F_0 \sin \Omega t$$

$$\eta^{st} = A \sin \Omega t \qquad \ddot{\eta} = -A\Omega^2 \sin \Omega t$$

$$(-1{,}3820\,\Omega^2 A + 12{,}687\,A)$$

$$(\cancel{1{,}3820A - 12{,}6687\,\Omega^2 A}) \sin \Omega t = F_0 \sin \Omega t \qquad \Omega^2 = 1{,}5^2\omega_1^2 =$$

$$= 2{,}25 \cdot 9{,}162 = 20{,}626$$

$$A = \cancel{\frac{F_0}{1{,}3826 - 12{,}6687\,\Omega^2}}$$

$$= 0{,}063144\,\frac{FL^3}{EI}$$

$$A_1 = \frac{F_0}{12{,}6687 - 1{,}3820\,\Omega^2} \cdot \frac{L^3}{EI} \qquad A_2 = \frac{F_0}{227{,}3313 - 3{,}6180\,\Omega^2} = 0{,}00659$$

$$A_1 = -0{,}063144 \frac{FL^3}{EI}$$

$$A_2 = 0{,}0065485 \frac{FL^3}{EI}$$

$$\underline{q} = \Phi \underline{\xi} = \begin{bmatrix} 1 & 1 \\ 0{,}61803 & -1{,}61803 \end{bmatrix} \begin{bmatrix} -0{,}063144 \\ 0{,}0065485 \end{bmatrix} \frac{FL^3}{EI} \sin \Omega t$$

$$= \begin{bmatrix} -0{,}0566 \\ -0{,}0496 \end{bmatrix} \frac{FL^3}{EI} \sin \Omega t$$

$\omega^2 = k/m$

$m = k/\omega^2$

$$\Omega = 0 \Rightarrow q_1 = 0{,}08333 \frac{FL^3}{EI}$$

$$q_2 = 0{,}0417 \frac{FL^3}{EI}$$

$m\ddot{x} + kx = F_0$

$\ddot{x} + \frac{k}{m}x = F_0/m$

$$\ddot{\eta}_1 + 2\zeta\omega_1\dot{\eta}_1 + 9{,}16693\,\eta_1 = \frac{F_0}{1{,}3820} \sin \Omega t$$

$$\ddot{\eta}_2 + 2\zeta\omega_2\dot{\eta}_2 + 62{,}8333\,\eta_2 = \frac{F_0}{3{,}6181} \sin \Omega t$$

$x_{st} = \frac{F_0}{k}$

$$A_1 = \frac{x_{st1}}{\sqrt{(\ ) + (\ )}}$$

$x_{st1} = \frac{F_0}{1{,}3820 \cdot \omega_1^2}$

$x_{st2} = \frac{F_0}{3{,}618 \cdot \cancel{22{,}733}\ 62{,}833}$

$$A_1 = \frac{x_{st1}}{\sqrt{(1-(\Omega/\omega_1)^2)^2 + (2\zeta\Omega/\omega)^2}} = \frac{1}{1{,}28549} x_{st1}$$

$\Omega = 1{,}5\omega_1$

$A_2 = \frac{1}{0{,}681439} x_{st2}$

$$A_1 = 0{,}06140\, F_0 \overbrace{\sin(\Omega t + \varphi_1)}^{= -(\ )}$$

$(\varphi_1 = 13{,}496^\circ)$ $\varphi_1 = -166{,}5^\circ$

$$A_2 = 0{,}006455 \sin(\Omega t + \varphi_2)$$

$\varphi_2 = -3{,}6807^\circ$

$$\underline{q} = \Phi \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = \begin{bmatrix} 1 & 1 \\ 0{,}61803 & -1{,}61803 \end{bmatrix} \begin{pmatrix} A_1 \\ A_2 \end{pmatrix}$$

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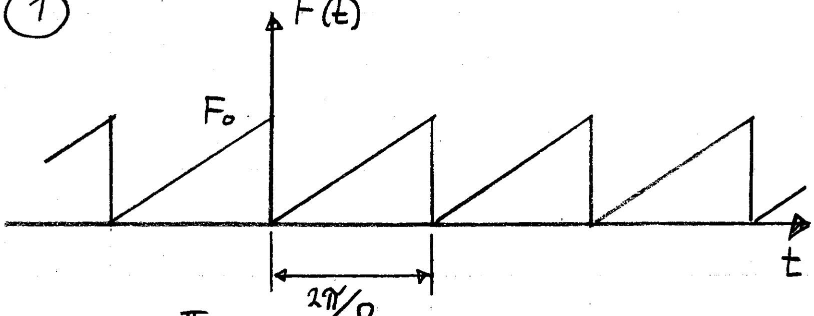


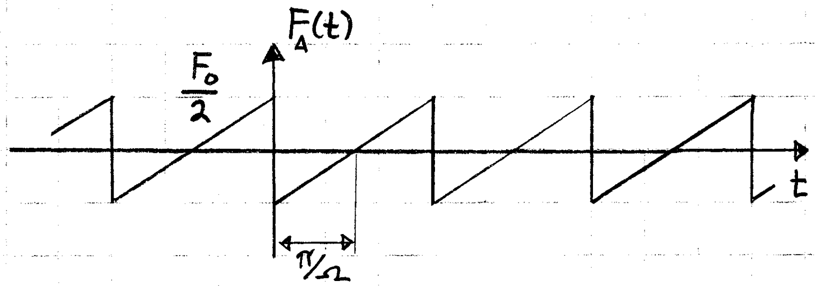
$$\Omega = 2\ \frac{1}{s}$$

$$\frac{\Omega}{\omega} = 0{,}80$$

$$a_\nu = \frac{\Omega}{\pi}\int_{-\pi/\Omega}^{\pi/\Omega} f(t)\cos(\nu\Omega t)\,dt \quad , \quad b_\nu = \frac{\Omega}{\pi}\int_{-\pi/\Omega}^{\pi/\Omega} f(t)\sin(\nu\Omega t)\,dt$$

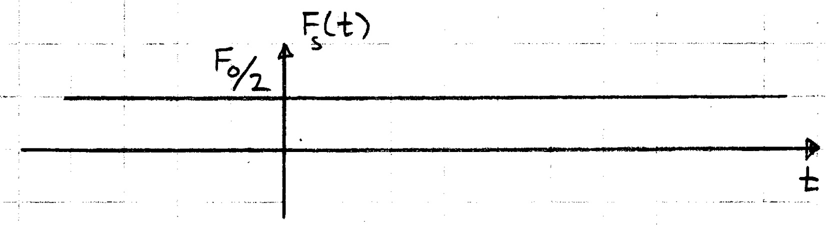
Heräte voidaan jakaa antimetriseen ja symmetriseen osaan:

antimetrinen osuus



$$f_A(t) = -1 + \frac{\Omega}{\pi}t$$

symmetrinen osuus



$$f_s(t) = 1$$

antimetrinen osuus:

$$a_\nu = \frac{\Omega}{\pi}\int_{-\pi/\Omega}^{\pi/\Omega} f_A(t)\cos(\nu\Omega t)\,dt = 0 \qquad \sin(\nu\pi) = 0$$

$$b_\nu = \frac{\Omega}{\pi}\int_{-\pi/\Omega}^{\pi/\Omega} f_A(t)\sin(\nu\Omega t)\,dt = \frac{2\Omega}{\pi}\int_0^{\pi/\Omega} f_A(t)\sin(\nu\Omega t)\,dt$$

$$= \frac{2\Omega}{\pi}\int_0^{\pi/\Omega}\left(-1 + \frac{\Omega}{\pi}t\right)\sin(\nu\Omega t)\,dt$$

$$b_\nu = \frac{2\Omega}{\pi}\left[\frac{1}{\nu\Omega}\int_0^{\pi/\Omega} -\nu\Omega \sin(\nu\Omega t)\,dt + \int_0^{\pi/\Omega} \frac{\Omega}{\pi}\underbrace{t}_{g}\underbrace{\sin(\nu\Omega t)}_{f'}\,dt\right]$$

osittaisintegrointi jälkimmäiselle integraalille

$$\int_a^b f'g\,dt = \Big/_a^b fg - \int_a^b g'f\,dt \qquad f = -\frac{1}{\nu\Omega}\cos(\nu\Omega t)\,;\ g = t$$

$$f' = \sin(\nu\Omega t) \qquad ;\ g' = 1$$

sin(νπ)=

$$b_\nu = \frac{2\Omega}{\pi}\left[\frac{1}{\nu\Omega}\Big/_0^{\pi/\Omega}\cos(\nu\Omega t) + \frac{\Omega}{\pi}\left(\Big/_0^{\pi/\Omega} -\frac{t}{\nu\Omega}\cos(\nu\Omega t) - \cancel{\int_0^{\pi/\Omega} -\frac{1}{\nu\Omega}\cos(\nu\Omega t)\,dt}\right)\right]$$

$$= \frac{2\Omega}{\pi}\left[\frac{1}{\nu\Omega}\big(\underbrace{-1-1}_{-2}\big) + \frac{\Omega}{\pi}\left(-\frac{\pi}{\nu\Omega^2}(-1+0)\right)\right]$$

$$= \frac{2\Omega}{\pi}\underbrace{\left[-\frac{2}{\nu\Omega} + \frac{1}{\nu\Omega}\right]}_{-\frac{1}{\nu\Omega}} = -\frac{2}{\nu\pi}$$

Fourier'n sarjakehitelmä (pruju s. 106)

$$F(t) = F_0\left[\frac{1}{2}a_0 + \sum_{\nu=1}^{\infty}\big(a_\nu\cos(\nu\Omega t) + b_\nu\sin(\nu\Omega t)\big)\right],\ \nu = 0,1,2,\dots$$

antimetrinen osuus

$$F_A(t) = -\frac{F_0}{2}\cdot\frac{2}{\pi}\sum_{\nu=1}^{\infty}\frac{1}{\nu}\sin(\nu\Omega t) = -\frac{F_0}{\pi}\sum_{\nu=1}^{\infty}\frac{1}{\nu}\sin(\nu\Omega t)$$

symmetrinen osuus

$$F_S(t) = \frac{F_0}{2}$$

$$\Rightarrow F(t) = F_S(t) + F_A(t) = F_0\left[\frac{1}{2} - \frac{1}{\pi}\sum_{\nu=1}^{\infty}\frac{1}{\nu}\sin(\nu\Omega t)\right],\ \nu = 1,2,3,\dots$$

Vaimentamattomalle värähtelijälle vasteen $x(t)$ harmonisten komponenttien vahvistuskertoimet V_ν ovat

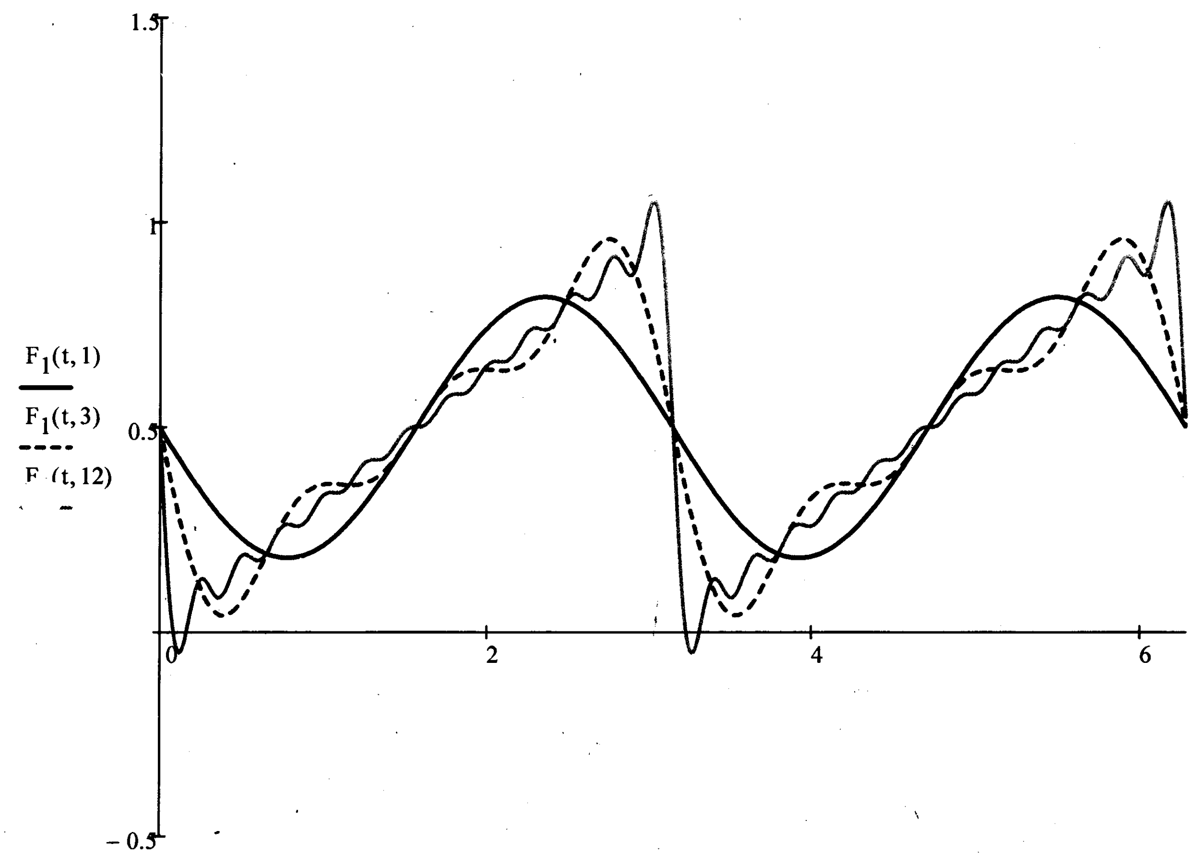
$$V_\nu = \frac{1}{1-\left(\frac{\nu\Omega}{\omega}\right)^2} \quad ; \quad \frac{\Omega}{\omega} = 0{,}80 \quad , \quad \Omega = 2\,\frac{1}{s}$$

$$\Rightarrow V_\nu = \frac{1}{1-0{,}64\nu^2} \; ; \qquad u(t) = u_{st}\left[\frac{1}{2} + \sum_{\nu=1}^{\infty} V_\nu b_\nu \sin(\nu\Omega t)\right]$$

$$\Rightarrow u(t) = u_{st}\left[\frac{1}{2} + \frac{1}{\pi}\sum_{\nu=1}^{\infty} \frac{1}{\nu(1-0{,}64\nu^2)} \sin(2\nu t)\right] \; , \quad \nu = 1,2,3,\ldots$$

$F_0 := 1 \qquad \Omega_1 := 0.80 \quad k := 1 \qquad u_{st} := \dfrac{F_0}{k} \qquad t := 0, 0.01 .. 2\cdot\pi$

$$F_1(t,n) := F_0 \cdot \left[\frac{1}{2} - \frac{1}{\pi} \cdot \sum_{\nu=1}^{n} \left(\frac{1}{\nu} \cdot \sin(2\nu\cdot t) \right) \right] \qquad u(t,n) := u_{st} \cdot \left[\frac{1}{2} - \frac{1}{\pi} \cdot \sum_{\nu=1}^{n} \left[\frac{1}{\nu\cdot\left(1 - 0.64\cdot\nu^2\right)} \cdot \sin(2\nu\cdot t) \right] \right]$$



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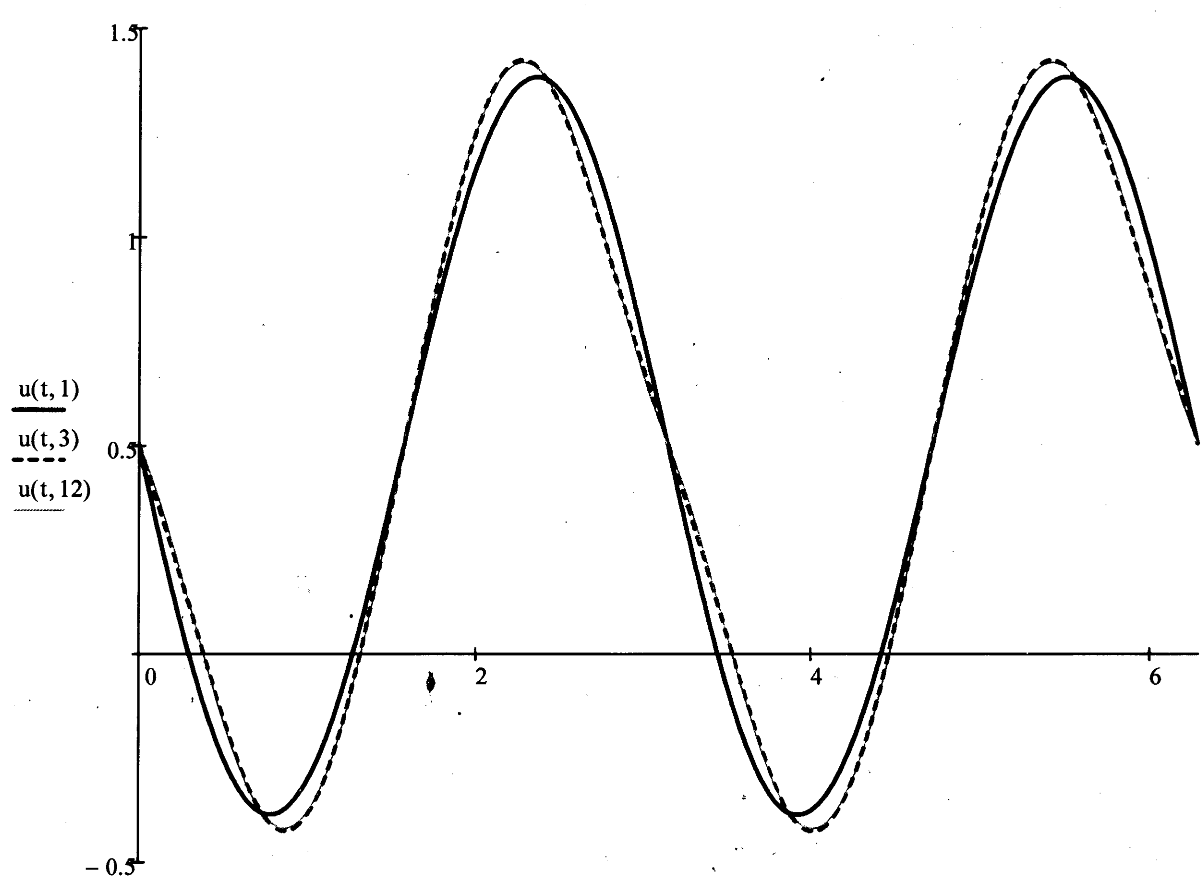

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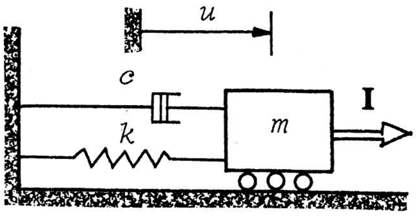

Kuvan vaimennettuun levossa olevaan systeemiin vaikuttaa hetkellä $t = 0$ impulssi $I = 1{,}0\,\mathrm{Ns}$. Määritä vaunun siirtymän suurin arvo. Mikä on siirtymän suurin arvo, jos vaimennusta ei ole? $k = 200\,\mathrm{N/m}$, $m = 3\,\mathrm{kg}$, $c = 15\,\mathrm{kg/s}$

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{200\,\mathrm{kg\,m/s^2m}}{3\,\mathrm{kg}}} \approx 8{,}165\ \frac{1}{\mathrm{s}}$$

$$c_{kr} = 2\sqrt{mk} = 2\sqrt{3\,\mathrm{kg}\cdot 200\,\mathrm{kgm/s^2m}} \approx 48{,}99\ \mathrm{kg/s}$$

$$\zeta = \frac{c}{c_{kr}} = \frac{15\,\mathrm{kg/s}}{48{,}99\,\mathrm{kg/s}} \approx 0{,}3062$$

$$\sqrt{1-\zeta^2} = \sqrt{1-0{,}3062^2} \approx 0{,}9520$$

$$\omega_d = \omega\sqrt{1-\zeta^2} = 8{,}165\ \frac{1}{\mathrm{s}}\cdot 0{,}9520 \approx 7{,}773\ \frac{1}{\mathrm{s}}$$

Impulssivaste: $u(t) = \dfrac{I}{m\omega_d}\, e^{-\zeta\omega t}\sin\omega_d t$

$$\frac{du}{dt} = \frac{I}{m\omega_d}\left[(-\zeta\omega\, \cancel{e^{-\zeta\omega t}}\sin\omega_d t + \cancel{e^{-\zeta\omega t}}\,\omega_d\cos\omega_d t\right] = 0$$

$$\Rightarrow\ -\zeta\omega\sin\omega_d t + \omega_d\cos\omega_d t = 0$$

$$\Rightarrow\ \tan\omega_d t_1 = \frac{\omega_d}{\zeta\omega} = \frac{\sqrt{1-\zeta^2}}{\zeta} = \frac{0{,}9520}{0{,}3062} \approx 3{,}109$$

$$\Rightarrow\ \omega_d t_1 = 1{,}260 \quad \Rightarrow\ t_1 = \frac{1{,}260}{7{,}773}\,\mathrm{s} \approx 0{,}1621\ \mathrm{s}$$

$$u_{max} = u(t_1) = \frac{I}{m\omega_d}\, e^{-\zeta\omega t_1}\sin\omega_d t_1$$

$$= \frac{1{,}0\ \frac{\cancel{\mathrm{kg}}\mathrm{m}}{\mathrm{s}^{\cancel{2}}}\,\cancel{\mathrm{s}}}{3\,\cancel{\mathrm{kg}}\cdot 7{,}773\,\frac{1}{\cancel{\mathrm{s}}}}\, e^{-0{,}3062\cdot 8{,}165\frac{1}{\mathrm{s}}\cdot 0{,}1621\,\mathrm{s}} \cdot \sin(1{,}260\ \mathrm{rad})$$

$$\approx 0{,}04288\cdot 0{,}6668\cdot 0{,}9521\ \mathrm{m} \approx 0{,}02722\ \mathrm{m}$$

$$\Rightarrow\ u_{max} \approx 27{,}2\ \mathrm{mm}$$

Ilman vaimennusta: $u_{max} = \hat{u} = \dfrac{I}{m\omega} = \dfrac{1}{3\cdot 8{,}165}\ \mathrm{m} \approx 0{,}0408\ \mathrm{m}$