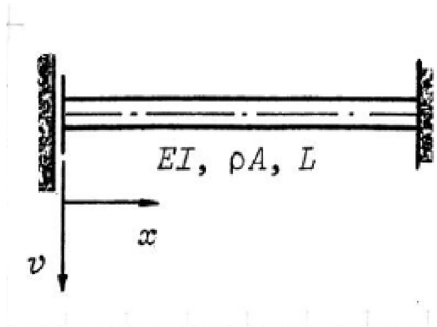
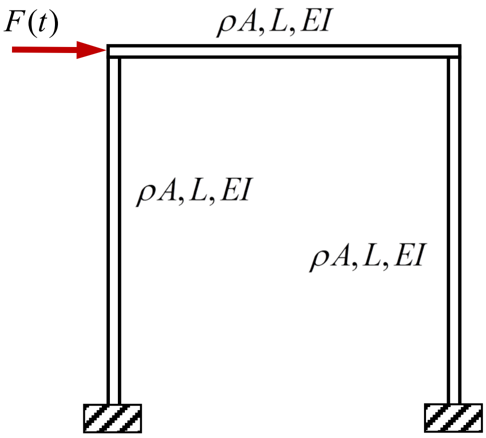


1. Determinate the stiffness matrix using two beam elements. Condense statically the rotational DOF. Determinate the lowest eigenpair by the inverse vector iteration. Use the lumped mass approximation. By Sturm's sequence rule find out whether the second eigenfrequency is higher than $\omega = 35\sqrt{EI/\rho AL^4}$.



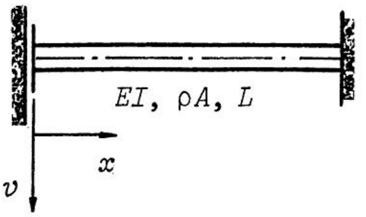
2. Use the eigenpair in Exercise 1 and determinate the steady state response when

$\rho A = 6{,}0$ kg/m, $\dfrac{EI}{L^3} = 1{,}346$ kN/m,

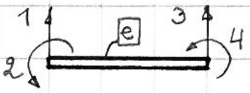
$L = 3$ m (IPE 80)

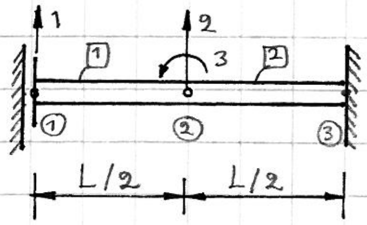
with the harmonic excitation

$F(t) = 200\sin(25 \cdot t)$ N .



Muodosta kuvan toisesta päästään jäykästi ja toisesta johteella tuetun tasapaksun ja homogeenisen palkin kahta elementtiä käyttävä laskentamalli. Tiivistä rotaatiovapausaste pois ja määritä palkin kaksi alinta ominaiskulmataajuutta. Käytä keskitettyä massamatriisia.





$$[k^1] = \frac{EI}{(L/2)^3}\begin{bmatrix} 12 & -12 & 6L/2 \\ & 12 & -6L/2 \\ \text{symm.} & & 4(L/2)^2 \end{bmatrix}$$

$$[k^2] = \frac{EI}{(L/2)^3}\begin{bmatrix} 12 & 6L/2 \\ & 4(L/2)^2 \end{bmatrix}$$

$$\Rightarrow \quad [k^1] = \frac{8EI}{L^3}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 12 & -12 & 3L \\ -12 & 12 & -3L \\ 3L & -3L & L^2 \end{bmatrix} \end{array}\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \qquad [k^2] = \frac{8EI}{L^3}\begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} 12 & 3L \\ 3L & L^2 \end{bmatrix} \end{array}\begin{array}{c} \\ 2 \\ 3 \end{array}$$

$$\Rightarrow \quad [K] = \frac{8EI}{L^3}\left[\begin{array}{cc|c} 12 & -12 & 3L \\ -12 & 24 & 0 \\ \hline 3L & 0 & 2L^2 \end{array}\right], \quad [M]_L = \frac{1}{4}\rho A L\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$[K^*] = [K_{11}] - [K_{12}][K_{22}]^{-1}[K_{21}]$$

$$= \left(\begin{bmatrix} 12 & -12 \\ -12 & 24 \end{bmatrix} - \begin{bmatrix} 3L \\ 0 \end{bmatrix}\cdot\frac{1}{2L^2}[3L \;\; 0]\right)\frac{8EI}{L^3} = \frac{8EI}{L^3}\begin{bmatrix} 15/2 & -12 \\ -12 & 24 \end{bmatrix}$$

$$\bar{\lambda} = \frac{1}{4}\frac{\rho A L}{8EI/L^3}\lambda, \quad \det\left(\begin{bmatrix} 15/2 & -12 \\ -12 & 24 \end{bmatrix} - \bar{\lambda}\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}\right) = 0$$

$$\Rightarrow \quad (15/2 - \bar{\lambda})(24 - 2\bar{\lambda}) - 144 = 0 \quad \Rightarrow \quad 2\bar{\lambda}^2 - 39\bar{\lambda} + 36 = 0$$

$$\Rightarrow \quad \bar{\lambda}_{1,2} \approx \begin{cases} 0{,}9715 \\ 18{,}53 \end{cases} \quad \Rightarrow \quad \lambda = 32\bar{\lambda}\cdot\frac{EI}{\rho A L^4}$$

$$\Rightarrow \quad \lambda_{1,2} = \begin{cases} 31{,}088 \\ 592{,}88 \end{cases}\frac{EI}{\rho A L^4} \quad \Rightarrow \quad \begin{cases} \omega_1 \approx 5{,}576\sqrt{EI/\rho A L^4} \\ \omega_2 \approx 24{,}35\sqrt{EI/\rho A L^4} \end{cases}$$

tarkka: $\omega_1 = 2{,}365^2\sqrt{} = 5{,}593\sqrt{EI/\rho A L^4}$

$\omega_2 = 5{,}498^2\sqrt{} = 30{,}228\sqrt{EI/\rho A L^4}$

Inverse matrix iteration

$\underline{K}^{-1} / \quad \underline{K}\,\underline{\phi} = \lambda \underline{M}\,\underline{\phi}$

$$\underline{\phi} = \lambda \underline{K}^{-1}\underline{M}\,\underline{\phi}$$

$$\frac{1}{\lambda}\underline{\phi} = \underline{K}^{-1}\underline{M}\,\underline{\phi}$$

$$\Rightarrow \frac{1}{\lambda}\underline{\phi}^{(1)} = \underline{K}^{-1}\underline{M}\,\underline{\phi}^{(0)}$$

The first trial $\underline{\phi}^{(0)} = \begin{pmatrix} 1 \\ 0,5 \end{pmatrix}$

$$\underline{K}^{-1}\underline{M}\,\underline{\phi}^{0} = \frac{8AL^4}{EI}\begin{pmatrix} 0,03125 \\ 0,016927 \end{pmatrix} = \frac{1}{32,00\ EI/8AL^4}\begin{pmatrix} 1 \\ 0,54167 \end{pmatrix}$$

$$\lambda^{(1)} = 32\ EI/8AL^4$$

$$\underline{\phi}^{0} = \begin{pmatrix} 1 \\ 0,54167 \end{pmatrix} \Rightarrow \underline{K}^{-1}\underline{M}\,\underline{\phi}^{0} = \frac{8AL^4}{EI}\begin{pmatrix} 0,032118 \\ 0,017470 \end{pmatrix} = \frac{1}{31,135}\begin{pmatrix} 1 \\ 0,5439 \end{pmatrix}$$

$$\underline{\phi}^{(0)} = \begin{pmatrix} 1 \\ 0,5439 \end{pmatrix} \qquad \underline{K}^{-1}\underline{M}\,\underline{\phi}^{(0)} = \begin{pmatrix} 0,03216 \\ 0,0174954 \end{pmatrix} = \frac{1}{31,097}\begin{pmatrix} 1 \\ 0,5440 \end{pmatrix}$$

$$\tilde{\lambda}_1 = 31,097\ EI/8AL^4 \qquad \underline{\phi}_1 = \begin{pmatrix} 1 \\ 0,5440 \end{pmatrix}$$

approximation for 1st eigenpair

Rayleigh quotient (Gives better approx.)

$$\lambda_R = \frac{\underline{\phi}^T \underline{K}\,\underline{\phi}}{\underline{\phi}^T \underline{M}\,\underline{\phi}} = 31,0872$$

$$\lambda_R = 31,087\ EI/8AL$$

$$\omega_1 = \sqrt{\lambda_R} = 5,576 \cdot \sqrt{EI/8AL^4}$$

Sturm's sequency rule

$\bar{\omega} = 35\sqrt{EI/\mathcal{S}AL^4}$ $\bar{\omega}^2 = 1225\ EI/\mathcal{S}AL^4$

$K = \frac{8EI}{L^3}\begin{bmatrix} 7,5 & -12 \\ -12 & 24 \end{bmatrix}$ $M_L = \mathcal{S}AL\begin{bmatrix} 0,25 & 0 \\ 0 & 0,5 \end{bmatrix}$

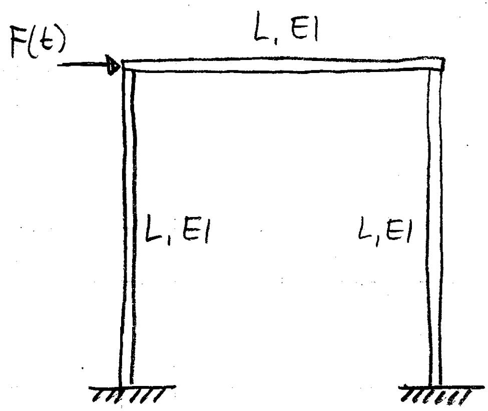
$K - \bar{\omega}^2 M = \frac{EI}{L^3}\begin{bmatrix} -246,25 & -96 \\ -96 & -420,5 \end{bmatrix} \cdot 96/246,25 \ \ominus$

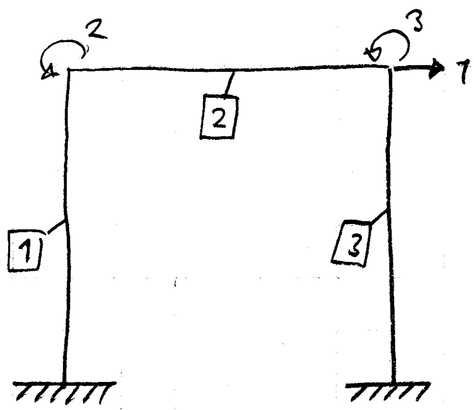
$\sim \begin{bmatrix} -246,25 & -96 \\ 0 & -420,5+96^2/246,25 \end{bmatrix}$

$\sim \begin{bmatrix} -246,25 & -96 \\ 0 & -383,07 \end{bmatrix}$

$\Rightarrow$ two negative values on diagonal
so two eigenvalues lower than $\bar{\omega}$

$\therefore \quad \omega_1 < \omega_2 < \bar{\omega}$





$$[M] = \rho A L \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad ; \quad [K] = \frac{EI}{L^3} \begin{bmatrix} 24 & 6L & 6L \\ 6L & 8L^2 & 2L^2 \\ 6L & 2L^2 & 8L^2 \end{bmatrix}$$

$$\det([K] - \lambda [M]) = 0 \qquad \lambda = \frac{\rho A L^4}{EI} \omega^2$$

$$\begin{vmatrix} 24 - 2\lambda & 6L & 6L \\ 6L & 8L^2 & 2L^2 \\ 6L & 2L^2 & 8L^2 \end{vmatrix} = 0$$

$$(24 - 2\lambda) 60 L^4 - 6L \cdot 36 L^3 + 6L \cdot (-36 L^3) = 0$$

$$1440 \cancel{L^4} - 120 \cancel{L^4} \lambda - 216 \cancel{L^4} - 216 \cancel{L^4} = 0$$

$$\lambda = 8{,}4$$

$$\Rightarrow \quad \omega = \sqrt{8{,}4} \sqrt{\frac{EI}{\rho A L^4}} \approx 25{,}062588$$

$$\rho A = 6 \frac{kg}{m} \quad ; \quad \frac{EI}{L^3} = 1{,}346 \frac{kN}{m} \quad ; \quad L = 3\,m \qquad \text{(IPE 80)}$$

③/1

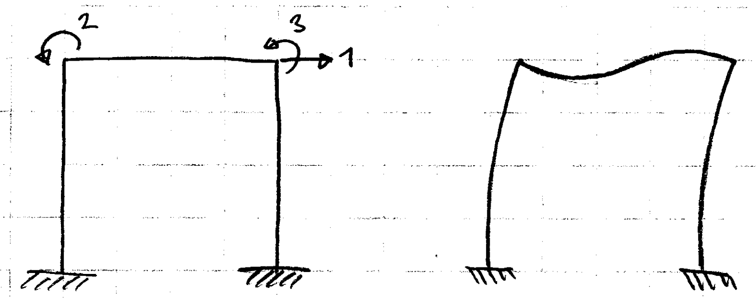
ominaismuoto: valitaan $\phi_1 = 1$

$$\begin{bmatrix} 8L^2 & 2L^2 \\ 2L^2 & 8L^2 \end{bmatrix} \begin{bmatrix} \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} -6L \\ -6L \end{bmatrix} \Rightarrow \begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix} \begin{bmatrix} \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} -6/L \\ -6/L \end{bmatrix}$$

$$\begin{bmatrix} \phi_2 \\ \phi_3 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 2 & 8 \end{bmatrix}^{-1} \begin{bmatrix} -6/L \\ -6/L \end{bmatrix} = \frac{1}{\underbrace{64-4}_{60}} \begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} -6/L \\ -6/L \end{bmatrix}$$

$$\begin{bmatrix} \phi_2 \\ \phi_3 \end{bmatrix} = \frac{1}{60} \begin{bmatrix} -36/L \\ -36/L \end{bmatrix} = -\frac{3}{5L} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -0{,}6/L \\ -0{,}6/L \end{bmatrix}$$

$$\underline{\phi} = \begin{bmatrix} 1 \\ -0{,}6/L \\ -0{,}6/L \end{bmatrix}$$



$$\hat{\underline{\phi}}_1 = \frac{1}{\sqrt{\underline{\phi}^T M \underline{\phi}^T}} \underline{\phi} = \frac{1}{\sqrt{2\rho AL}} \begin{bmatrix} 1 \\ -0{,}6/L \\ -0{,}6/L \end{bmatrix}$$

$\rho AL = 18\,kg$

$$\mathcal{F} = \hat{\phi}_1^T \begin{bmatrix} F_1(t) \\ 0 \\ 0 \end{bmatrix} = \frac{1}{\sqrt{2\rho AL}} F_1(t) = \frac{1}{\sqrt{2\rho AL}} \cdot 200 \sin(25t)\ N$$

$$= \frac{100}{3} \sin(25 \cdot t)\ N$$

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$$V = \frac{1}{1-\left(\frac{\Omega}{\omega}\right)^2} = \frac{1}{1-\left(\frac{25}{25{,}062588}\right)^2} = 200{,}469108$$

$$\ddot{\eta}_1 + \lambda \eta_1 = \frac{100}{3} \sin(25\,t) \quad ; \quad \lambda = 8{,}4 \frac{EI}{\varrho A L^4} = 628{,}73 \frac{1}{s^2}$$

$$[\ddot{\eta}] = \sqrt{kg}\,\frac{m}{s^2} \qquad [\eta] = \sqrt{kg}\; m \qquad [\mathcal{F}] = \sqrt{kg}\,\frac{m}{s^2}$$

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$$\ddot{x} + \omega^2 x = \hat{F}(t) \sin(25t) \qquad \hat{F} = \frac{100}{3}$$

$$\Rightarrow \quad \hat{F} = x_{st}\,\omega^2 \qquad x_{st} = \frac{\hat{F}}{\omega^2} = 0{,}053067 \sqrt{kg}\; m$$

$$\underline{q}_{st} = \underline{\hat{\phi}}\, x_{st} = \frac{1}{\underbrace{\sqrt{2\varrho A L}}_{18kg}} \begin{bmatrix} 1 \\ -0{,}6/L \\ -0{,}6/L \end{bmatrix} \cdot 0{,}053067 \cancel{\sqrt{kg}}\; m$$

$$\underline{q}_{st} = \begin{bmatrix} 8{,}8445\ mm \\ -1{,}7685 \cdot 10^{-3} \\ -1{,}7685 \cdot 10^{-3} \end{bmatrix}$$

$$\Omega = 25\ rad/s$$

$$\underline{q} = \underline{\hat{\phi}}\, \frac{x_{st}}{1-\left(\frac{\Omega}{\omega}\right)^2} \sin(\Omega t)$$

$$= \underline{q}_{st}\, V \sin(\Omega t) \qquad V = 200{,}46$$

$$= \begin{bmatrix} 1{,}772\ m \\ -0{,}354\ rad \\ -0{,}354\ rad \end{bmatrix} \sin(\Omega t)$$

3/3