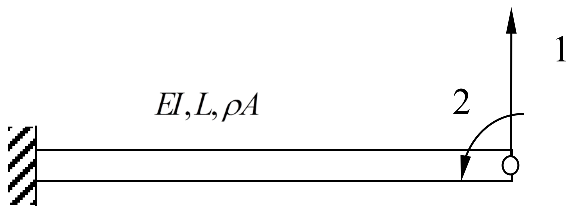

1. Determinate the stiffness matrix and consistent mass matrix of the cantilever beam.
   - Evaluate eigenvalues of the the system.
   - Evaluate lowest eigenvalue by the inverse vector iteration with the starting vector $\boldsymbol{\phi} = \begin{pmatrix} 1 & 1/L \end{pmatrix}^T$.
   - At every iteration evaluate the estimate for eigenvalue by Rayleigh's quotient .
   - Repeat calculations by shifting $\omega_0 = 35\sqrt{EI/\rho A L^4}$

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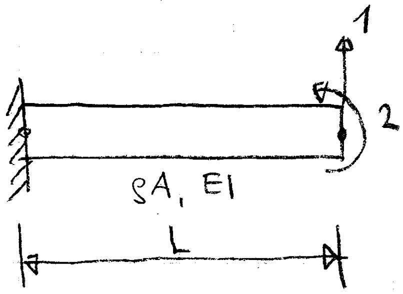

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix}$$

$$[M] = \frac{\rho AL}{420}\begin{bmatrix} 156 & -22L \\ -22L & 4L^2 \end{bmatrix}$$

$$\bar{\lambda} = \frac{\rho A L^4}{420 EI}\lambda$$

$$\det([\bar{K}] - \bar{\lambda}[\bar{M}]) = \begin{vmatrix} 12 - 156\bar{\lambda} & -6L + 22L\bar{\lambda} \\ -6L + 22L\bar{\lambda} & 4L^2 - 4L^2\bar{\lambda} \end{vmatrix} = 0$$

$$\Rightarrow \quad 35\bar{\lambda}^2 - 102\bar{\lambda} + 3 = 0$$

$$\Rightarrow \quad \bar{\lambda}_{1,2} = \frac{102 \pm \sqrt{102^2 - 4\cdot 3\cdot 35}}{2\cdot 35} \approx \frac{102 \pm 99{,}9199680}{70}$$

$$\Rightarrow \quad \bar{\lambda}_{1,2} = \begin{cases} 0{,}02971474 \\ 2{,}8845710 \end{cases}$$

$$\Rightarrow \quad \lambda_{1,2} = \frac{420 EI}{\rho A L^4}\bar{\lambda}_{1,2} = \begin{cases} 12{,}4801908 \dfrac{EI}{\rho A L^4} \\ 1211{,}5198 \dfrac{EI}{\rho A L^4} \end{cases}$$

$$\Rightarrow \quad \omega_{1,2} = \sqrt{\lambda_{1,2}} = \begin{cases} 3{,}5327 \sqrt{\dfrac{EI}{\rho A L^4}} \\ 34{,}8069 \sqrt{\dfrac{EI}{\rho A L^4}} \end{cases}$$

$$K = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} \qquad \omega_1 = 3{,}5327\sqrt{EI/\rho A L^4}$$

$$\omega_2 = 34{,}8069\sqrt{EI/\rho A L^4}$$

$$M = \frac{\rho A L}{420}\begin{bmatrix} 156 & -22L \\ -22L & 4L^2 \end{bmatrix}$$

$$(K - \omega^2 M)\underline{\phi} = 0 \qquad \omega^2 = \lambda\, EI/\rho A L^4$$

$$\left(K - \lambda \frac{EI}{\rho A L^4} M\right)\underline{\phi} = \underline{0}$$

$$\left(\frac{EI}{L^3}\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} - \lambda \frac{EI}{\rho A L^4}\frac{\rho A L}{420}\begin{bmatrix} 156 & -22L \\ -22L & 4L^2 \end{bmatrix}\right)\underline{\Phi} = \underline{0}$$

$$\left(\begin{bmatrix} 12 & -6L \\ -6L & 4L^2 \end{bmatrix} - \lambda \begin{bmatrix} 156 & -22L \\ -22L & 4L^2 \end{bmatrix}\right)\underline{\Phi} = 0$$

$$\underline{\phi}^{(0)} = \begin{pmatrix} 1 \\ 1/L \end{pmatrix}$$

$$\underline{\phi}^{(1)} = K^{-1}M\underline{\phi}^{(0)} = \begin{pmatrix} 0{,}0845 \\ 0{,}1167 \end{pmatrix} \qquad \lambda^{(1)} = \frac{1}{0{,}0845} = 11{,}77$$

$$\hat{\lambda}_R^{(1)} = \frac{\phi^{(1)T} K \phi^{(1)}}{\phi^{T(1)} M \phi^{(1)}} = 12{,}4804$$

$$\phi^{(2)} = K^{-1}M\phi^{(1)} = \begin{pmatrix} 0{,}0802 \\ 0{,}1104 \end{pmatrix} \qquad \hat{\lambda}^{(2)} = \frac{1}{0{,}0802} = 12{,}473$$

$$\hat{\lambda}_R^{2} = 12{,}4802 \qquad \lambda = \lambda_0 + \hat{\lambda}$$

$$\omega_0 = 35\sqrt{EI/\rho A L^4} \qquad \lambda_0 = 1225$$

$$K^* = K - 1225 \cdot M = \begin{bmatrix} -443 & 58{,}1667 \\ 58{,}1667 & -7{,}667 \end{bmatrix}$$

$$\phi^0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad \phi^{(1)} = \begin{pmatrix} 0{,}0036 \\ 0{,}0330 \end{pmatrix} \qquad \hat{\lambda}^{(1)} = 277{,}0 \qquad \phi^2 = \begin{pmatrix} -0{,}052 \\ -0{,}7022 \end{pmatrix} \qquad \hat{\lambda}^2 = 10{,}8\ldots$$

$$\lambda_R^{(0)} = 14{,}48 \qquad \lambda_R^{(1)} = 1129$$

$$33{,}6$$

Shift $\omega_0 = 35\sqrt{EI/SAL^4}$

$$\underline{K}\,\underline{\phi} = (\lambda + \omega_0^2)\,\underline{M}\,\underline{\phi}$$

$\lambda = \bar{\lambda} + \omega_0^2$

$\omega = \sqrt{\lambda}$

$$(\underline{K} - \omega_0^2 \underline{M})\,\underline{\phi} = \bar{\lambda}\,\underline{M}\,\underline{\phi}$$

$$\underline{K}^* = \underline{K} - \omega_0^2 \underline{M} = \frac{EI}{L^3}\begin{bmatrix} -443 & 58{,}167L \\ 58{,}167L & -7{,}667L^2 \end{bmatrix}$$

Starting vector $\underline{\phi}^0 = \begin{pmatrix} 1 \\ 1/L \end{pmatrix}$

$\bar{\omega}^2 = \bar{\lambda}$

$$\underline{\phi}^{(1)} = \underline{K}^{*-1}\underline{M}\,\underline{\phi}^{(0)} = \begin{pmatrix} 0{,}0036097 \\ 0{,}0329576 \end{pmatrix} = \frac{1}{277{,}8}\begin{pmatrix} 1 \\ 9{,}1356/L \end{pmatrix}$$

$\omega^{(1)} = 38{,}75\sqrt{\ }$

$$\lambda_R^{(1)} = \frac{\underline{\phi}^{(1)T}\underline{K}\,\underline{\phi}^{(1)}}{\underline{\phi}^{(1)T}\underline{M}\,\underline{\phi}^{(1)}} = 1129{,}0\,\frac{EI}{SAL^4} \qquad \omega_R^{(1)} = 33{,}60\sqrt{\frac{EI}{SAL^4}}$$

$\bar{\lambda}$

$$\underline{\phi}^{(2)} = \underline{K}^{*-1}\underline{M}\,\underline{\phi}^{(1)} = \begin{pmatrix} -0{,}09196 \\ -0{,}70219 \end{pmatrix} = \frac{1}{-10{,}875}\begin{pmatrix} 1 \\ 7{,}636/L \end{pmatrix}$$

$$\lambda^{(2)} = \bar{\lambda} + \omega_0^2 = 1214{,}1 \qquad \omega^{(2)} = 34{,}844\sqrt{EI/SAL^4}$$

$$\lambda_R^{(2)} = \frac{\underline{\phi}^{(2)T}\underline{K}\,\underline{\phi}^{(2)}}{\underline{\phi}^{2T}\underline{M}\,\underline{\phi}^{2}} = 1211{,}509 \qquad \omega_R^{(2)} = 34{,}807\sqrt{\frac{EI}{SAL^4}}$$

$\bar{\lambda}$

$$\underline{\phi}^{(3)} = \begin{pmatrix} -0{,}07434 \\ -0{,}56668/L \end{pmatrix} = \frac{1}{-13{,}451}\begin{pmatrix} 1 \\ 7{,}6226 \end{pmatrix}$$

$$\lambda^{(3)} = \bar{\lambda} + \omega_0^2 = 1211{,}55 \qquad \omega^{(3)} = 34{,}8073\sqrt{\frac{EI}{SAL^4}}$$

$$\lambda_R^{(3)} = \frac{\underline{\phi}^{(3)T}\underline{K}\,\underline{\phi}^{(3)}}{\underline{\phi}^{(3)T}\underline{M}\,\underline{\phi}^{(3)}} = 1211{,}5198 \qquad \omega_R^{(3)} = 34{,}8069\sqrt{\frac{EI}{SAL^4}}$$