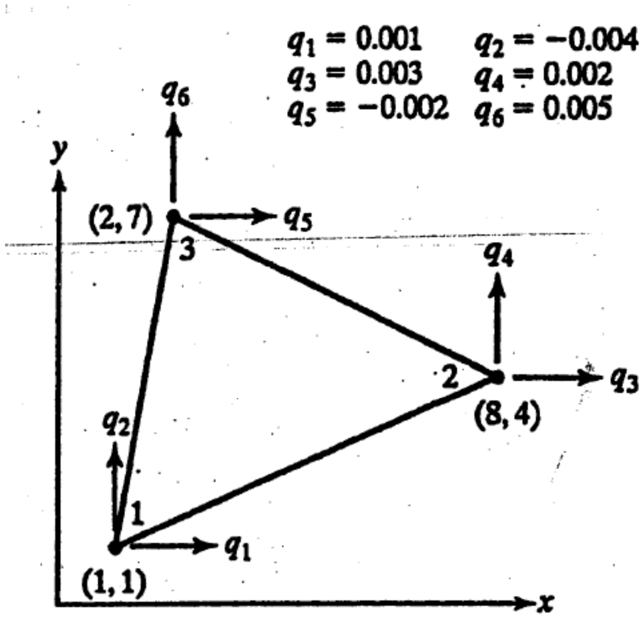
Tampereen teknillinen yliopisto
Teknisen suunnittelun laitos

EDE-21100 Elementtimenetelmän perusteet. Harjoitus 9 Syksy 2013.

1. For the triangular element shown in figure below, obtain the strain-displacement relation matrix **B** and determine the strains $\varepsilon_x, \varepsilon_y$ and γ_{xy}.



Note: **q and x have the same units.**

$$\mathbf{B} = \frac{1}{\det \mathbf{J}} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$\mathbf{J} = \begin{bmatrix} x_{13} & y_{13} \\ x_{23} & y_{23} \end{bmatrix}$$

Node	x	y
1	1	1
2	8	4
3	2	7

y_23	-3	x_32	-6
y_31	6	x_13	-1
y_12	-3	x_21	7

q

0.001
-0.004
0.003
0.002
-0.002
0.005

J

-1	-6
6	-3

det[**J**]

39

B

-0.07692	0	0.153846	0	-0.07692	0
0	-0.15385	0	-0.02564	0	0.179487
-0.15385	-0.07692	-0.02564	0.153846	0.179487	-0.07692

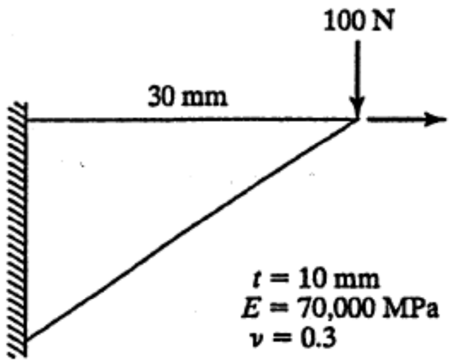
0.000538
0.001462
-0.00036

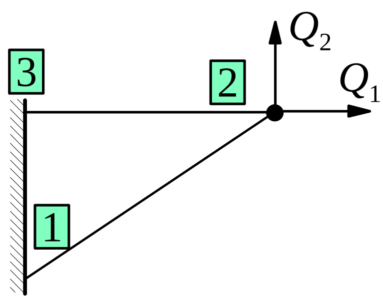
strains **B** * **q**

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \mathbf{B} \cdot \mathbf{q}$$

5.9. For the configuration shown in Fig. P.5.9, determine the deflection at the point of load application using a one-element model. If a mesh of several triangular elements is used, comment on the stress values in the elements close to the tip.

$$\mathbf{D}=\frac{E}{1-\upsilon^2}\begin{bmatrix}1 & \upsilon & 0\\ \upsilon & 1 & 0\\ 0 & 0 & (1-\upsilon)/2\end{bmatrix}$$





$$\mathbf{k}_e = t\,A\mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}$$

$$\mathbf{J}=\begin{bmatrix}x_{13} & y_{13}\\ x_{23} & y_{23}\end{bmatrix}=\begin{bmatrix}0 & -20\\ 30 & 0\end{bmatrix}mm \Rightarrow |\mathbf{J}| = 600mm^2$$

$$\mathbf{B}^{\mathrm{T}}\mathbf{D}\mathbf{B}=[\mathbf{B}_1 \quad \mathbf{B}_2 \quad \mathbf{B}_3]^{\mathrm{T}}\mathbf{D}[\mathbf{B}_1 \quad \mathbf{B}_2 \quad \mathbf{B}_3]$$

$$=[\mathbf{B}_1 \quad \mathbf{B}_2 \quad \mathbf{B}_3]^{\mathrm{T}}[\mathbf{D}\mathbf{B}_1 \quad \mathbf{D}\mathbf{B}_2 \quad \mathbf{D}\mathbf{B}_3]$$

$$=\begin{array}{c}\begin{bmatrix}\mathbf{B}_1^{\mathrm{T}}\mathbf{D}\mathbf{B}_1 & \mathbf{B}_1^{\mathrm{T}}\mathbf{D}\mathbf{B}_2 & \mathbf{B}_1^{\mathrm{T}}\mathbf{D}\mathbf{B}_3\\ \mathbf{B}_2^{\mathrm{T}}\mathbf{D}\mathbf{B}_1 & \mathbf{B}_2^{\mathrm{T}}\mathbf{D}\mathbf{B}_2 & \mathbf{B}_2^{\mathrm{T}}\mathbf{D}\mathbf{B}_3\\ \mathbf{B}_3^{\mathrm{T}}\mathbf{D}\mathbf{B}_1 & \mathbf{B}_3^{\mathrm{T}}\mathbf{D}\mathbf{B}_2 & \mathbf{B}_3^{\mathrm{T}}\mathbf{D}\mathbf{B}_3\end{bmatrix}\end{array}\begin{array}{l}1-2\\ 3-4\\ 5-6\end{array}$$

$$\mathbf{B}_2=\frac{1}{|\mathbf{J}|}\begin{bmatrix}y_{31} & 0\\ 0 & x_{13}\\ x_{13} & y_{31}\end{bmatrix}=\frac{1}{600mm}\begin{bmatrix}20 & 0\\ 0 & 0\\ 0 & 20\end{bmatrix}$$

$$\mathbf{K}=t\,A\mathbf{B}_2^{\mathrm{T}}\mathbf{D}\mathbf{B}_2 = 10\,mm\cdot 300mm^2\,\frac{70000N}{mm^2\left(1-0.3^2\right)}\mathbf{B}_2^{\mathrm{T}}\mathbf{D}\mathbf{B}_2$$

$$=10\cdot 300mm^3\,\frac{70000N}{mm^2\left(1-0.3^2\right)}\cdot\frac{mm^2}{600^2mm^4}\begin{bmatrix}20 & 0 & 0\\ 0 & 0 & 20\end{bmatrix}\begin{bmatrix}1 & 0.3 & 0\\ 0.3 & 1 & 0\\ 0 & 0 & 0.35\end{bmatrix}\begin{bmatrix}20 & 0\\ 0 & 0\\ 0 & 20\end{bmatrix}$$

$$=641.0256\begin{bmatrix}400 & 0\\ 0 & 140\end{bmatrix}\frac{N}{mm}$$

$$\begin{bmatrix}Q_1\\ Q_2\end{bmatrix}=\mathbf{K}^{-1}\mathbf{F}=\mathbf{K}^{-1}\begin{bmatrix}50\\ -100\end{bmatrix}N=\begin{bmatrix}0.195\\ -1.114\end{bmatrix}10^{-3}mm$$

Kun elementtijakoa tihennetään, niin jännitys nousee pistevoiman kohdalla vastaavasti, joten pistevoima pitäisi korvata painejakautumalla. Laitan tämän tiedoston loppuun vielä vanhan (viime vuosituhannelta) olevan ratkaisun

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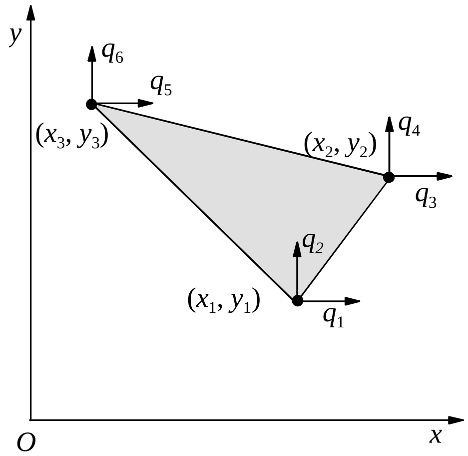
EDE-21100 Elementtimenetelmän perusteet. Harjoitus 9 Syksy 2013.

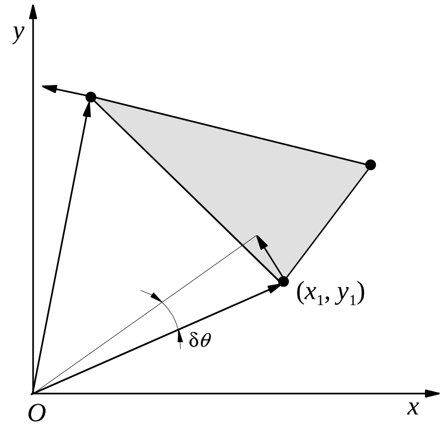
1. Kuvan CST-elementtiin kohdistuu pieni rotaatio $\delta\theta$ origon ympäri. Osoita, ettei elementin kimmoenergia muutu tästä jäykän kappaleen liikkeestä.

$$\mathbf{B} = \frac{1}{\det \mathbf{J}} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$



Muodostetaan kunkin solmun siirtymävektori, joka aiheutuu rotaatiosta origon ympäri, erikseen. Rotaatiovektori

$$\delta\mathbf{\Theta} = \begin{bmatrix} 0 \\ 0 \\ \delta\theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \delta\theta$$



ja solmun 1 siirtymävektori

$$\mathbf{q}_1 = \delta\mathbf{\Theta} \times \mathbf{r}_1 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 0 & 1 \\ x_1 & y_1 & 0 \end{vmatrix} \delta\theta = \begin{bmatrix} -y_1 \\ x_1 \\ 0 \end{bmatrix} \delta\theta$$

jättämällä z-komponentti pois ja laskemalla vastaavat siirtymät solmuille 2 ja 3 saadaan

$$\mathbf{q} = \begin{pmatrix} -y_1 & x_1 & -y_2 & x_2 & -y_3 & x_3 \end{pmatrix}^{\mathrm{T}} \delta\theta$$

Lasketaan sitten venymäkomponentit

$$\boldsymbol{\varepsilon} = \frac{1}{\det \mathbf{J}} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix} \begin{bmatrix} -y_1 \\ x_1 \\ -y_2 \\ x_2 \\ -y_3 \\ x_3 \end{bmatrix} \delta\theta$$

$$= \frac{1}{\det \mathbf{J}} \begin{bmatrix} -(y_2 - y_3) y_1 - (y_3 - y_1) y_2 - (y_1 - y_2) y_3 \\ (x_3 - x_2) x_1 + (x_1 - x_3) x_2 + (x_2 - x_1) x_3 \\ -(x_3 - x_2) y_1 + (y_2 - y_3) x_1 - (x_1 - x_3) y_2 + (y_3 - y_1) x_2 - (x_2 - x_1) y_3 + (y_1 - y_2) x_3 \end{bmatrix} \delta\theta$$

$$= \frac{1}{\det \mathbf{J}} \begin{bmatrix} y_3 y_1 - y_2 y_1 + y_1 y_2 - y_3 y_2 + y_2 y_3 - y_1 y_3 \\ x_3 x_1 - x_2 x_1 + x_1 x_2 - x_3 x_2 + x_2 x_3 - x_1 x_3 \\ -x_3 y_1 + x_2 y_1 + y_2 x_1 - y_3 x_1 - x_1 y_2 + x_3 y_2 + y_3 x_2 - y_1 x_2 - x_2 y_3 + x_1 y_3 + y_1 x_3 - y_2 x_3 \end{bmatrix} \delta\theta$$

$$= \frac{1}{\det \mathbf{J}} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \delta\theta$$

joten pieni jäykän kappaleen rotaatio $\delta\theta$ origon ympäri ei aiheuta elementtiin venymää, eikä siten myöskään kimmoenergiaa.

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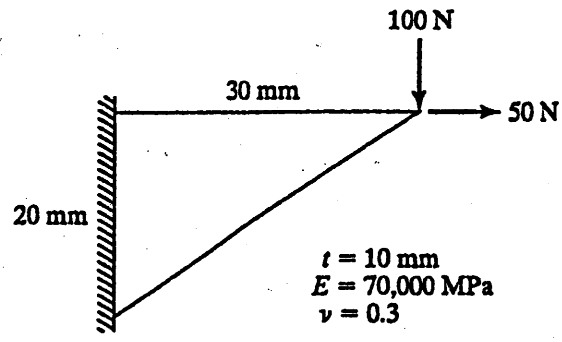
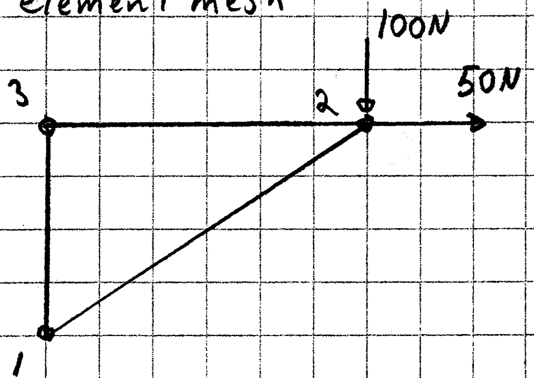


FIGURE P5.9

(mm, N)

element mesh



node coordinates

$x_1 = 0 \qquad y_1 = 0$

$x_2 = 30 \qquad y_2 = 20$

$x_3 = 0 \qquad y_3 = 20$

$$[J] = \begin{bmatrix} x_{13} & y_{13} \\ x_{23} & y_{23} \end{bmatrix} = \begin{bmatrix} 0 & -20 \\ 30 & 0 \end{bmatrix} \qquad \det[J] = 600 = 2A$$

$$[B] = \frac{1}{\det[J]} \begin{bmatrix} y_{23} & 0 & y_{31} & 0 & y_{12} & 0 \\ 0 & x_{32} & 0 & x_{13} & 0 & x_{21} \\ x_{32} & y_{23} & x_{13} & y_{31} & x_{21} & y_{12} \end{bmatrix}$$

$$[B] = \frac{1}{600} \begin{bmatrix} 0 & 0 & 20 & 0 & -20 & 0 \\ 0 & -30 & 0 & 0 & 0 & 30 \\ -30 & 0 & 0 & 20 & 30 & -20 \end{bmatrix}$$

$$[B] = \frac{1}{60} \begin{bmatrix} 0 & 0 & 2 & 0 & -2 & 0 \\ 0 & -3 & 0 & 0 & 0 & 3 \\ -3 & 0 & 0 & 2 & 3 & -2 \end{bmatrix}$$

$$[D] = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1}{2}(1-\nu) \end{bmatrix} = \frac{70000}{0.91} \begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0 \\ 0 & 0 & 0.35 \end{bmatrix}$$

stiffness matrix $[K] = tA\,[B]^T[D][B]$

$$[D][B] = \frac{7000}{0.91\cdot 6}\begin{bmatrix} 0 & -0.9 & 2 & 0 & -2 & 0.9 \\ 0 & -3 & 0.6 & 0 & -0.6 & 3 \\ -1.05 & 0 & 0 & 0.7 & 1.05 & -0.7 \end{bmatrix}$$

$$[K] = \frac{10\cdot 300\cdot 7000}{60\cdot 0.91\cdot 6}\begin{bmatrix} 0 & 0 & -3 \\ 0 & -3 & 0 \\ 2 & 0 & 0 \\ 0 & 0 & 2 \\ -2 & 0 & 3 \\ 0 & 3 & -2 \end{bmatrix}\begin{bmatrix} 0 & -0.9 & 2 & 0 & -2 & 0.9 \\ 0 & -3 & 0.6 & 0 & -0.6 & 3 \\ -1.05 & 0 & 0 & 0.7 & 1.05 & -0.7 \end{bmatrix}$$

$$[K] = 64102{,}56\begin{bmatrix} 3.15 & 0 & 0 & -2.1 & -3.15 & 2{,}1 \\ 0 & 9 & -1{,}8 & 0 & 1.8 & -9 \\ 0 & -1{,}8 & 4 & 0 & -4 & 1{,}8 \\ -2{,}1 & 0 & 0 & 1{,}4 & 2.1 & -1{,}4 \\ -3.15 & 1.8 & -4 & 2{,}1 & 7.15 & -3{,}9 \\ 2{,}1 & -9 & 1.8 & -1.4 & -3.9 & 10.4 \end{bmatrix}$$

$$\{F\} = [0 \quad 0 \quad 50 \quad -100 \quad 0 \quad 0]^T$$

boundary conditions $Q_1 = Q_2 = Q_5 = Q_6 = 0$

elimination method $[K] = 64102.56\begin{bmatrix} 4 & 0 \\ 0 & 1.4 \end{bmatrix}$

$$64102.56\begin{bmatrix} 4 & 0 \\ 0 & 1.4 \end{bmatrix}\begin{bmatrix} Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 50 \\ -100 \end{bmatrix}$$

$$\begin{bmatrix} Q_3 \\ Q_4 \end{bmatrix} = \begin{bmatrix} 0.195\cdot 10^{-3} \\ -1.114\cdot 10^{-3} \end{bmatrix} mm = \begin{bmatrix} 195\,\mu m \\ -1114\,\mu m \end{bmatrix}$$