Introduction to materials modelling

14. exercise - numerical solution

1. A metallic bar is loaded by a sinusoidally time-varying strain

$$\varepsilon(t) = \varepsilon_{\mathrm{a}} \sin(2\pi t/t_{\mathrm{per}}),$$

where t_{per} is the period of a cycle. It is assumed that the material obeys the following constitutive model where creep and damage evolutions are coupled

$$\begin{aligned}
\sigma &= (1-D)E(\varepsilon - \varepsilon^{\mathrm{c}}), \\
\dot{\varepsilon}^{\mathrm{c}} &= \frac{h(T)}{t_{\mathrm{c}}} \left(\frac{|\sigma|}{(1-D)\sigma_{\mathrm{r}}} \right)^{p} \frac{\sigma}{|\sigma|}, \\
\dot{D} &= \frac{h(T)}{t_{\mathrm{d}}} \left(\frac{|\sigma|}{(1-D)\sigma_{\mathrm{r}}} \right)^{p},
\end{aligned}$$

where $h(T) = \exp(-Q/RT)$, Q is the activation energy, R is the gas constant and T is the absolute temperature. Investigate the material behaviour when $\varepsilon_{\mathrm{a}} = 0.005$ and temperature is 700°C. Compute with two maximum strain rate values 10^{-3} 1/s and 10^{4} 1/s. Determine the period t_{per} such that it results the required strain rate. The material parameters have values $E = 100$ GPa, $\sigma_{\mathrm{r}} = 75$ MPa, $p = 6$ and $t_{\mathrm{c}} = 100$ s, $t_{\mathrm{d}} = 10$ s. For the thermal activation energy you can use the value 150 kJ/mol.

Deaw the damage evolution as a function of cycles. How many cycles does the material sustain before failure? Draw also the stress-strain curve at every one hundredth cycle when the maximum strain arte is 10^{-3} 1/s and every tenth cycle in the case of 10^{-4} 1/s.

Hint. Code a simple program which integrates the material model e.g. by using the explicit Euler method. Remember that the explicit Euler method is only conditionaly stable. Thus keep the time step Δt below the critical time step Δt_{cr}. Investigate the critical time step as a function of stress level.