Introduction to materials modelling

13. exercise - numerical solution

1. Solve the uniaxial creep problem

$$\sigma = E(\varepsilon - \varepsilon_{\rm c}), \tag{1}$$

where the creep strain-rate is governed by equation

$$\dot{\varepsilon}_{\rm c} = \frac{1}{\tau_{\rm pr}}\left(\frac{\sigma}{\sigma_{\rm r}}\right).$$

The pseudo-relaxation time $\tau_{\rm pr}$ is a material constant and $\sigma_{\rm r}$ is an arbitrary reference stress. The relaxation time $\tau_{\rm r}$ is $\tau_{\rm r} = \tau_{\rm pr}\varepsilon_{\rm r} = \tau_{\rm pr}\sigma_{\rm r}/E$. Consider a constant strain-rate loading $\varepsilon(t) = \varepsilon_{\rm r}t/\tau_{\rm r}$. Integrate the stress response to the time instant $4\tau_{\rm r}$ using an appropriate time-step Δt. Use (a) explicit Euler and (b) implicit Euler method.

Hint. Formulate equation (1) in a non-dimensional form using a non-dimensional stress $y = \sigma/\sigma_{\rm r}$. First determine the critical time-step for the explicit Euler method.

2. Consider the nonlinear kinematic Armstrong-Frederick (Chaboche model with one back stress) type hardening model, which can be described as

$$\boldsymbol{\sigma} = \boldsymbol{C}^{\rm e}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\rm p}), \tag{2}$$

$$f(\boldsymbol{\sigma}, \boldsymbol{X}) = \sigma_{\rm eff} - \sigma_{\rm y0} = 0, \tag{3}$$

$$\sigma_{\rm eff} = \sqrt{\tfrac{3}{2}(\boldsymbol{s} - \boldsymbol{X}) : (\boldsymbol{s} - \boldsymbol{X})}, \tag{4}$$

$$\dot{\boldsymbol{\varepsilon}}^{\rm p} = \dot{\lambda}\frac{3}{2}\frac{\boldsymbol{s} - \boldsymbol{X}}{\sigma_{\rm eff}}, \tag{5}$$

$$\dot{\boldsymbol{\alpha}} = \dot{\boldsymbol{\varepsilon}}^{\rm p} - \dot{\lambda}\gamma\boldsymbol{\alpha} \tag{6}$$

where $\boldsymbol{s}$ is the deviatoric stress $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}{\rm tr}(\boldsymbol{\sigma})\boldsymbol{I}$ and the relation between the internal variable $\boldsymbol{\alpha}$ and the back stress $\boldsymbol{X}$ is $\boldsymbol{X} = \frac{2}{3}C\boldsymbol{\alpha}$. Material parameters are $\sigma_{\rm y0}, C, \gamma$. Notice that $\dot{\lambda}$ equals to the rate of the effective plastic strain

$$\dot{\bar{\varepsilon}}^{\rm p} = \sqrt{\frac{2}{3}\dot{\boldsymbol{\varepsilon}}^{\rm p} : \dot{\boldsymbol{\varepsilon}}^{\rm p}}.$$

- Compute the maximum stress predicted by the model using monotonous uniaxial tensile stress as loading.
- Determine the stress response in the monotonous unaxial tensile stress loading, i.e. the $\varepsilon^{\rm p} - \sigma$-curve.

Hint. Remember that $\boldsymbol{\varepsilon}^{\rm p}, \boldsymbol{\alpha}$ and $\boldsymbol{X}$ are deviatoric, so they can be treated as a functions of only one component.