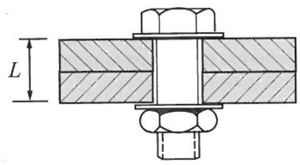
Introduction to materials modelling

12. exercise - creep models

1. The cross-sectional area of a bolt shown below is A. In the bolt there is a pretension σ_0. Plates which the bolt connects can be assumed to be rigid. Assume a non-linear power law creep model (Norton-Bailey) for the bolt

$$\dot{\varepsilon} = E^{-1}\dot{\sigma} + \frac{1}{t_\mathrm{c}}\left(\frac{\sigma}{\sigma_\mathrm{ref}}\right)^p.$$

Determine the duration in which the stress is decreased to one half of the initial pretension stress. Values for the material parameters are $E = 200$ GPa, $\sigma_0 = 100$ MPa, $\sigma_\mathrm{ref} = 100$ MPa, $p = 4$, $t_\mathrm{c} = 10^4$ h.



2. Investigate the behaviour of the non-linear power law type creep model (Norton-Bailey)

$$\dot{\boldsymbol{\varepsilon}}^\mathrm{c} = \frac{1}{t_\mathrm{c}}\left(\frac{\bar{\sigma}}{\sigma_\mathrm{r}}\right)^p \frac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}}, \tag{1}$$

where t_c and p are model parameters and σ_r is an arbitrary reference stress (drag stress). The von Mises effective stress is denoted as $\bar{\sigma} = \sqrt{3J_2}$, where J_2 is the second invariant of the deviatoric stress tensor. The parameter t_c has a dimension of time. Determine the relaxation time τ. Solve the stress relaxation problem when a tensile specimen is stretched to a strain ε_0 parallel to the specimen. Draw stress-time relaxation crves for the cases $\varepsilon_0 = \varepsilon_\mathrm{r}$ and $\varepsilon_0 = \frac{1}{2}\varepsilon_\mathrm{r}$ and use the following values for p: $p = 1, 3, 5$ ($\varepsilon_\mathrm{r} = \sigma_\mathrm{r}/E$, where E is the Young's modulus).

3. Investigate further the power law type creep model, but now with hardening. The evolution equation for the creep strain rate is

$$\dot{\boldsymbol{\varepsilon}}^\mathrm{c} = \frac{1}{t_\mathrm{c}}\left(\frac{\bar{\sigma}}{\sigma_0 + K}\right)^p \frac{\partial\bar{\sigma}}{\partial\boldsymbol{\sigma}},$$

where σ_0 is the intial yield stress, t_c time parameter (relatex to relaxation time) and p dimensionless parameter and the hardening rule is assumed to be of a saturation type

$$K = K_\infty(1 - \exp(-h\bar{\varepsilon}^\mathrm{c}/K_\infty)).$$

The effective creep strain is defined as

$$\dot{\bar{\varepsilon}}^\mathrm{c} = \sqrt{\tfrac{2}{3}\dot{\varepsilon}^\mathrm{c}_{ij}\dot{\varepsilon}^\mathrm{c}_{ij}}, \qquad \bar{\varepsilon}^\mathrm{c} = \int_0^t \dot{\bar{\varepsilon}}^\mathrm{c}\,\mathrm{d}t.$$

The effective stress is the convetional von Mises stress $\bar{\sigma} = \sqrt{3J_2}$. Notice that in a monotonous uniaxial stress process the effective creep strain equals to the creep strain in the direction of the stress (or its abolut value if compressive loading).

Investigate the creep behaviour of the model with different values of p as $p = 1, 2, 4$ under uniaxial stress $\sigma_{11} = \sigma$. The elastic model is linear and isotropic having

Young's modulus E. Use the stress values $\sigma = \frac{1}{2}\sigma_0$ and σ_0. Assume the following ratios between the material parameters: $K_\infty = \sigma_0, h = E/50$ and $E/\sigma_0 = 500$.

You can solve the system e.g. by the explicit Euler method, where the first order ordinary differential equation $\dot{y} = f(y)$ is replaced by difference approximation

$$\frac{y_{n+1} - y_n}{\Delta t} = f(y_n),$$

where $y_n = y(t_n)$ is the known state at time instant t_n and the solution is looked for time $t_{n+1} = t_n + \Delta t$. Remember that the explicit Euler method is conditionally stable, thus the time step has to be smaller than the critical time step.