Introduction to materials modelling

9. exercise – yield conditions

1. A long thin walled cylindrical steel tube with a diameter D and thickness t is loaded by internal overpressure p_1 and external pressure p_2. Assume that the external pressure does not influence to the axial stress and that $p_2 = rp_1, r \geq 0$. The tube is yielding when $p_1 = p_0$ and $p_2 = 0$.

   (a) Determine the internal pressure $p_1 = p_\mathrm{y}$, when the tube is yielding when $r > 0$. Use both the Tresca and von Mises yield conditions.

   (b) Draw the (p_y, r)-relationship for both yield conditions.

   (c) Find the value of r at which the limit pressures predicted by the two criteria have the largest difference and give the reason. Assume that the both yield conditions have been calibrated to coincide in pure shear. Draw the yield surfaces on the deviatoric plane.

2. A material is yielding in uniaxial tension at the stress level f_t and in uniaxial compression at stress level $-f_\mathrm{c}$. What is the yield stress in pure shear if the Drucker-Prager yield condition is used.

3. Consider the Dricker-Prager yield condition: $f(I_1, J_2) = \sqrt{3J_2} + \alpha I_1 - \beta = 0$. Show that the largest ratio between the compressive and tensile yield stresses $m = f_\mathrm{c}/f_\mathrm{t}$, where the model still predicts a finite biaxial compressive stress f_bc, i.e. when $\sigma_1 = \sigma_2 = -f_\mathrm{bc}$ is

   (a) in plane stress $m = 3$,

   (b) in plane strain $m = 3/(1 + 6\nu)$, where ν is the Poisson's ratio.

4. For certain polymers the following yield condition can be used

$$\sqrt{\sigma_\mathrm{e}^2 + \alpha^2(\sigma_\mathrm{m} - A)^2} - B = 0,$$

   where $\sigma_\mathrm{e} = \sqrt{3J_2} = \sqrt{\frac{3}{2}s_{ij}s_{ji}}$ and $\sigma_\mathrm{m} = \frac{1}{3}I_1 = \frac{1}{3}\sigma_{kk}$ and the deviatoric stress tensor is defined as $s_{ij} = \sigma_{ij} - \sigma_\mathrm{m}\delta_{ij}$. Determine the material parameters α, A and B when experiments give the uniaxial compressive stress σ_c, hydrostatic compressive stress p_c and the hydrostatic tensile stress p_t. Sketch the yield surface in the meridian plane $(\sigma_\mathrm{m}, \sigma_\mathrm{e})$. Sketch the yield surface also on the deviatoric plane.