Introduction to materials modelling

2. exercise – stress state, principal stresses, invariants

1. In a certain cross-section of a 3D beam, the stress tensor in the cartesian coordinate system (x_1, x_2, x_3) has the following form

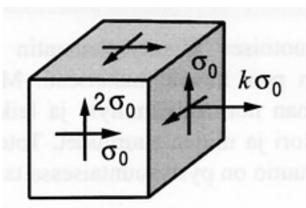
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_0 & -\sigma_0 & \sigma_0 \\ -\sigma_0 & 0 & 0 \\ \sigma_0 & 0 & 0 \end{bmatrix},$$

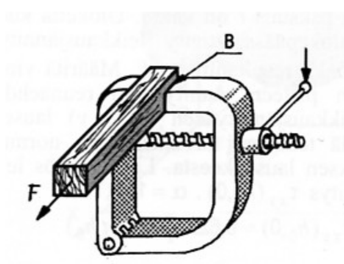
where σ_0 is some positive stress value.

   (a) Draw the stress element.
   (b) Determine the traction vector $\boldsymbol{t}$ on a plane with normal direction $(4, 3, 0)$.
   (c) Determine the normal and shear components in that plane.
   (d) Determine pricipal stresses.
   (e) Determine the stress deviator $\boldsymbol{s} = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})\boldsymbol{I}$.
   (f) Calculate the second invariant of the deviatoric stress $J_2 = \frac{1}{2}\mathrm{tr}(\boldsymbol{s}^2) = \frac{1}{2}s_{ij}s_{ji}$, and the effective von Mises stress $\sigma_{\mathrm{eff}} = \sqrt{3J_2}$.

2. Determine the stress tensor (matrix) $\boldsymbol{\sigma}$, the deviatoric stress tensor $\mathbf{s} = \boldsymbol{\sigma} - \frac{1}{3}\mathrm{tr}(\boldsymbol{\sigma})\mathbf{I}$ and the second invariant of the deviatoric stress $J_2 = \frac{1}{2}s_{ij}s_{ji}$ for the stress state shown below on the lhs.

   Determine the value of k such that the stress state corresponts to the state of plane stress. Determine for this stress state the principal stresses and the unit normal vector of the stress free surface. Calculate also the maximum shear stress.





3. A bar with a solid square cross section (height 20 mm) is subjected to a tensile force $F = 6$ kN, see the figure above on the rhs. The contact are of the screw press is 500 mm^2 and the coefficient of friction between the jaws and the bar is 0,3. Calculate the stress components of the stress tensor at the point B and determine also the principal stresses and directions at the moment of sliding. Calculate also the deviatoric stress tensor and its invariants J_2 ja J_3 and the Lode angle θ on the deviatoric plane.

4. A biaxial stress state (plane stess) is often used in determining the material parameters in the constitutive models. The stress state in the coordinate system alighned with the loading is thus

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \alpha\sigma_0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

where α is a dimensionless parameter, which depending on the test in question can have values in the range $(-1, 1)$. Determine as a function of the parameter α the following:

(a) the mean stress σ_{m},
(b) the effective stress $\sigma_{\mathrm{e}} = \sqrt{3J_2}$,
(c) Lode angle θ (see the defintion from the lecture notes)
(d) maximum shear stress τ_{max},
(e) and the normal direction of the plane where the maximum shear stress occurs.

What kind of stress state is the one when $\alpha = -1$?