Introduction to materials modelling

1. exercise – index notation

1. Exercising index notation and Einstein's summation convention.
   (a) Write out $\delta_{im}A_{mj}$ and express the result without the dummy index m.
   (b) Investigate if the following expressions are correct.
      i. $a_m b_s = c_m(f_r + d_r)$,
      ii. $a_m b_s = c_m(f_s + d_s)$,
      iii. $a_i = b_j c_i d_i$,
      iv. $x_i x_i = r^2$,
      v. $a_i b_j c_j = 3$.
   (c) Simplify the expression $\delta_{ij}\delta_{jk}\delta_{kp}\delta_{pi}$.
   (d) If $A_{ij} = -A_{ji}$, show that: $A_{ij}v_i v_j = 0$.
   (e) If $A_{ij} = -A_{ji}$ and $B_{ij} = B_{ji}$, show that: $A_{ij}B_{ij} = 0$.

2. Second order cartesian tensor A_{ij} can be expressed as teh following matrix

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 0 \\ 4 & 4 & 1 \\ -2 & 2 & 5 \end{bmatrix}$$

   Determine the values of the the following expressions:

   (a) A_{ii}, (b) $A_{ij}A_{ij}$, (c) $A_{ij}A_{ji}$, (c) $\delta_{ii}A_{mm}$, (d) $A_{pq}A_{pq}$,
   (e) $A_{ij} = \frac{1}{2}(A_{ij} + A_{ji})$, (f) $A_{ij} = \frac{1}{2}(A_{ij} - A_{ji})$.

3. How many equations are present in the following expressions. Write the equations out explicitly.

   (a) $v_m = Q_{mn}u_n$, (b) $A_{ij} = B_{ir}C_{rj}$, (c) $S = v_i Q_{ij} v_j$.