FEM advanced course

10. exercise – time integration methods - 1-D bar example

1. Two-step method of Lees is rather popular in solving transient heat transfer problem although it is notoriously oscillatory. The method can be written as

$$\boldsymbol{M}\frac{\boldsymbol{u}_{n+1}-\boldsymbol{u}_{n-1}}{2\Delta t}+\boldsymbol{K}\frac{\boldsymbol{u}_{n+1}+\boldsymbol{u}_{n}+\boldsymbol{u}_{n-1}}{3}=\boldsymbol{f}_n.$$

   (a) Investigate the stability properties of this method applied to a scalar heat conduction equation $\dot{y}+\lambda y=0$. Show that the method is unconditionally stable, but it can yield oscillatory solutions. **Hint.** Substitute $y_n=r^n$ and form the quadratic characteristic polynomial. Deduce that the absolute values of the roots are always smaller than one (or the modulus is smaller than one in the case of complex roots). Plot the roots or (the modulus) as a function of $\lambda\Delta t$.

   (b) Solve the 1-D heat conduction equation

$$\rho c\dot{u}-ku''=f=\text{constant},\quad u(0,t)=u(L,t)=0,$$

   using the method of Lees using time-step $\Delta t=\Delta t_{\text{cr}}$[1]. Since the method is a two-step method, integrate the first time-step using the implicit Euler method. How small should the time-step be in order to avoid oscillations. What are your conclusions. Use one quadratic element.

2. Solve the 1-D impact problem

$$\rho A\ddot{u}-EAu''=0$$

   with initial conditions $u(x,0)=\dot{u}(x,0)=0$ and clamped in one end $u(0,t)=0$ and with sudden application of a normal force F at $t=0$ at the free end $x=L$. Write a small code and use 20 linear elements and lumped mass matrix and the central difference method to integrate the discrete equations of motion. Use time step which is as close as possible to the critical time-step. Remember that an upper bound of the critical time step can be obtained from the single element eigenfrequency as $\Delta t_{\text{cr}}\le 2/\omega_{\max}$, where ω_{cr} is the maximum frequency of the elementwise eigenvalue problem $\boldsymbol{K\phi}=\omega^2\boldsymbol{M\phi}$.

Home exercise 10. Solve the problem 2 using geometrically non-linear model, i.e.

$$\rho A\ddot{u}-N'=0,$$

where the normsl force is given as $N=EA\varepsilon$, where ε is the Green-Lagrange strain $\varepsilon=u'+\frac{1}{2}(u')^2$. Experiment with different values of the force F. What are your conclusions?

Solution should be returned prior 15th of April

[1]The critical time step of the explicit Euler method.