FEM advanced course

1. exercise – intro to non-linear problems, Newton's method

1. In the section 2.1.2 of the study book (Wriggers: Nonlinear finite element methods) the equilibrium equation (2.8) can be written as

$$f(x;\hat{F}) = 2x\left(1 - \frac{1}{\sqrt{1+x^2}}\right) - \hat{F} = 0,$$

where $x = w/\ell$ is the non-dimensional deflection (state variable) and $\hat{F} = F/c\ell$ is the non-dimensional load (control variable). Formulate and code the Newton iteration process. Experiment the code with different values of $\hat{F}$ as 0.25, 0.5 and 1. Start with $x = 0.1$. Why the iteration cannot be started from the undeformed state $x = 0$?

2. Consider the same problem as in the previous exercise problem 2 but now having a non-linear force-elongation relation

$$S_F = c_0 \tanh(c_1\varepsilon/c_0), \qquad \varepsilon = \sqrt{1+x^2} - 1.$$

The equilibrium equation is

$$2S_F \sin\varphi = F, \qquad \sin\varphi = \frac{x}{\sqrt{1+x^2}},$$

which can be written as

$$f(x;\hat{F}) = 0,$$

where the non-dimensional force is defined as $\hat{F} = F/c_0$. Use $c_0 = 1, c_1 = 8$ and use the initial value $x_0 = 0.1$. Experiment $\hat{F} = 0.1, 0.2, 0.25$ and 0.3. What are your conclusions and explanations for the behaviour.

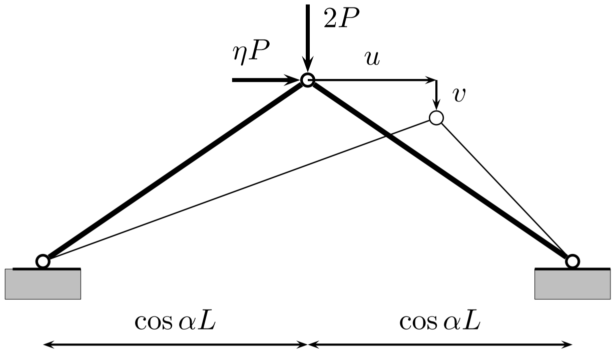
Can you figure out the reason for different performance of the Newton's method in these two problems?

3. Program a simple solver for non-linear algebraic equations and test it with the following problem:

$$f_1(q_1,q_2) = q_1^2 + q_2^2 - 1 = 0,$$
$$f_1(q_1,q_2) = q_1 - q_2 = 0,$$

i.e. yu have to find the crossing point of the circle and a line. Select a proper initial value. Design the code in such a way that it can be easily extended to future problems, including the present home assignment.

Home assignment 1. Consider a two-bar Mises truss. Length and the initial angle of the bars at the initial state are L and α, respectively, and the axial stiffness equals to EA. The bars are assumed to be absolutely rigid in bending. See, lecture notes *Computational Techniques for the non-linear analysis of structures*, Example 1.3.1.



The equilibrium equations can be written in a dimensionless form as

$$f_1(q_1, q_2; \lambda) = 2(\cos^2\alpha)q_1 + q_1^3 - 2(\sin\alpha)q_1q_2 + q_1q_2^2 - \eta\lambda = 0, \tag{1}$$

$$f_2(q_1, q_2; \lambda) = -(\sin\alpha)q_1^2 + q_1^2q_2 + 2(\sin^2\alpha)q_2 - 3(\sin\alpha)q_2^2 + q_2^3 - 2\lambda = 0, \tag{2}$$

where q_1, q_2 are the dimensionless horizontal and vertical displacements of the load point $q = v/L$, respectively, and the load parameter is $\lambda = P/EA$.

Write a computer code to solve the system incrementally-iterative, i.e. increment the load in steps $\lambda_n = \lambda_{n-1} + \Delta\lambda$ and correct the solution within a step with Newton's method. Design the code such that both full Newton's or a modfied Newton's method can be used. Compute the derivative needed in the Newton's method (the Jacobian) either by using explicit formula or by numerical differentiation.

Select the angle $\alpha = 30°$ and compute the equilibrium path up to $\lambda_{\max} = 0.19\sin^3\alpha$ using

1. a small load step, e.g $\Delta\lambda = \lambda_{\max}/100$ and $\eta = 0$,
2. large load step, e.g. $\Delta\lambda = \lambda_{\max}/2$ and $\Delta\lambda = \lambda_{\max}$ and $\eta = 0$,
3. with $\eta = 0.2$ what is the largest load for which you can find a converged solution?

Try to solve the equilibrium position of the symmetric deformation mode ($\eta = 0$) at $\lambda = \frac{\sqrt{3}}{9}\sin^3\alpha \approx 0.1925\sin^3\alpha$ using only a single load step.

Monitor and plot the convergence of the iterative procedure using both the residual $\|\boldsymbol{f}\|$ or the relative displacement $\|\delta\boldsymbol{q}_i\|/\|\Delta\boldsymbol{q}_i\|$. What are your conclusions about the convergence?

Solution report should be returned in Moodle prior to exercise 2