Continuum mechanics

1. exercise – mathematical preliminaries

1. (Holzapfel ex.9 p. 20)
   Find the axial vector of a skew tensor $\mathbf{W} = \frac{1}{2}(\mathbf{u} \otimes \mathbf{v} - \mathbf{v} \otimes \mathbf{u})$.

2. (CMT, ex 2.13 p. 68)
   Consider the dyad $\mathbf{D} = \mathbf{a} \otimes \mathbf{a}$ constructed from the vector $\mathbf{a}$.

   (a) Write out the components of $\mathbf{D}$ in matrix form.

   (b) Compute the three principal invariants of $\mathbf{D} : I_1, I_2, I_3$. Simplify your expressions as much as possible.

   (c) Compute the eigenvalues of $\mathbf{D}$.

3. Assume that $\mathbf{v}$ is an arbitrary vector and $\hat{\mathbf{n}}$ is a unit vector. Show that

   $$\mathbf{v} = (\mathbf{v} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}} + \hat{\mathbf{n}} \times (\mathbf{v} \times \hat{\mathbf{n}}).$$

   What is the meaning of this formula?

4. Derive the vector identity below connecting three arbitrary vectors $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$ by the method of vector analysis

   $$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}.$$