Simulation and Experimental Validation of Chaotic Behavior of the Airflow in a Ventilated Room

Jos van Schijndel

Technische Universiteit Eindhoven
University of Technology

Where innovation starts

Introduction

Chaos theory and what should we learn from it?

- **Can you solve this ?** $x_{n+1} = r x_n (1 - x_n)$

TU/e Technische Universiteit Eindhoven University of Technology

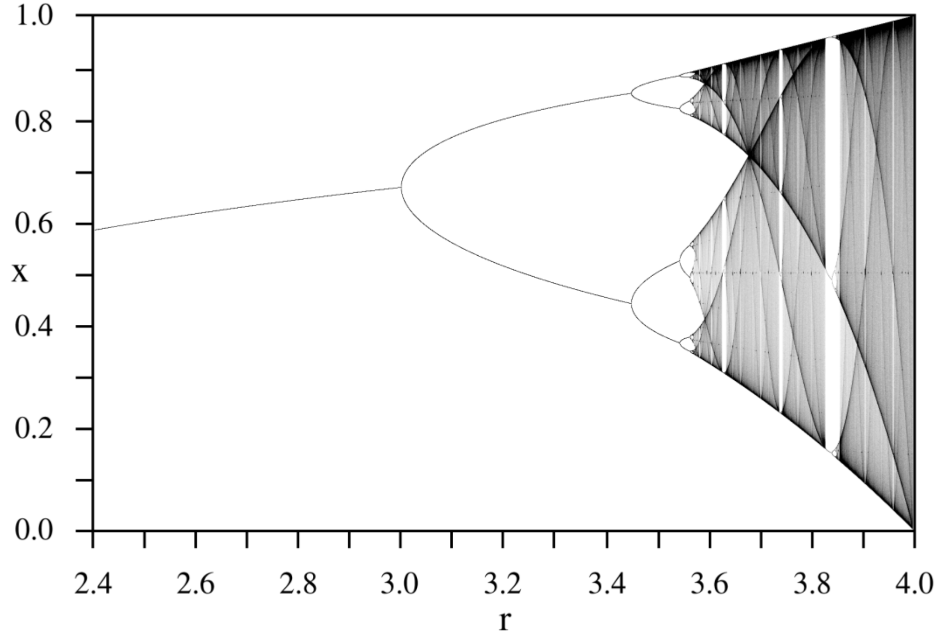
Chaos theory and what should we learn from it?

- **NO!**

- **Bifurcation diagram of the logistic map x → r x (1 – x).**
- **Each vertical slice shows the attractor for a specific value of r.**
- **The diagram displays period-doubling as r increases, eventually producing chaos**

- **Message:**
- **Seemingly simple systems can have very complex (chaotic) behaviors**

$$x_{n+1} = r x_n (1 - x_n)$$



Chaos theory and what should we learn from it?

- **What about this ?**
  - **These ODEs represent the simplified equations of convection rolls arising in the equations of the atmosphere.**
  - **Fully deterministic**
  - **i.e. a conceptual CFD model**

$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

$$\frac{dz}{dt} = xy - \beta z$$

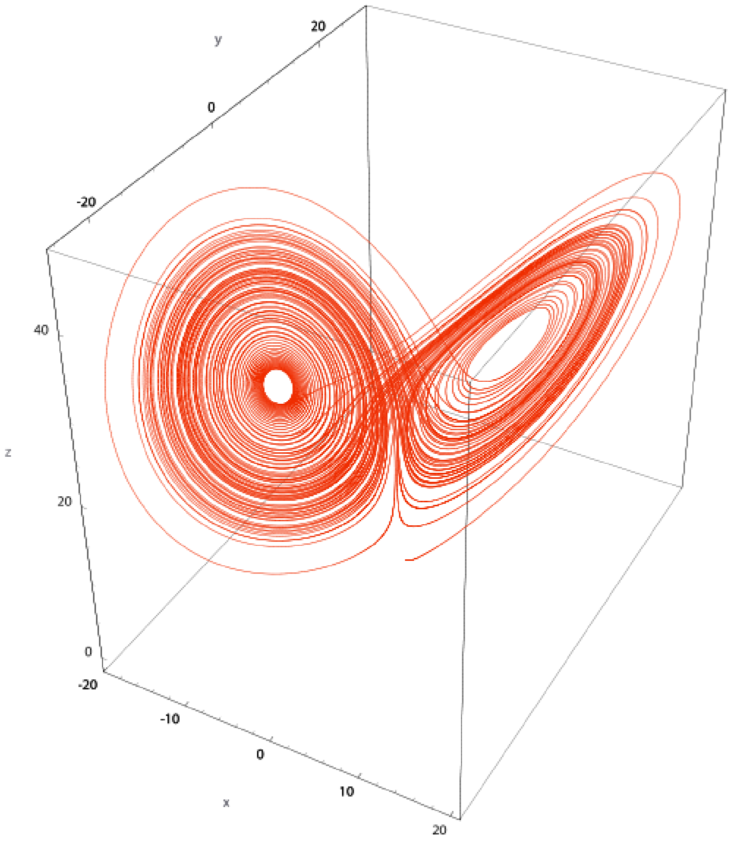
TU/e Technische Universiteit Eindhoven University of Technology

Chaos theory and what should we learn from it?

- **Solvable but:**

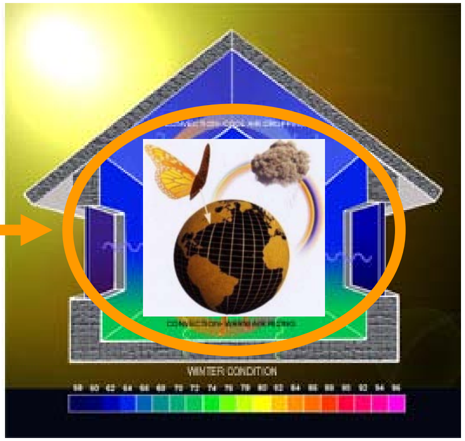
- **The Lorenz model has important implications for climate and weather prediction.**
- **The model is an explicit statement that atmospheres may exhibit a variety of quasi-periodic regimes that are, although fully deterministic, subject to abrupt and seemingly random change**

- **Message:**
- **Sensitive dependence on the initial condition and parameters**



Technische Universiteit Eindhoven University of Technology

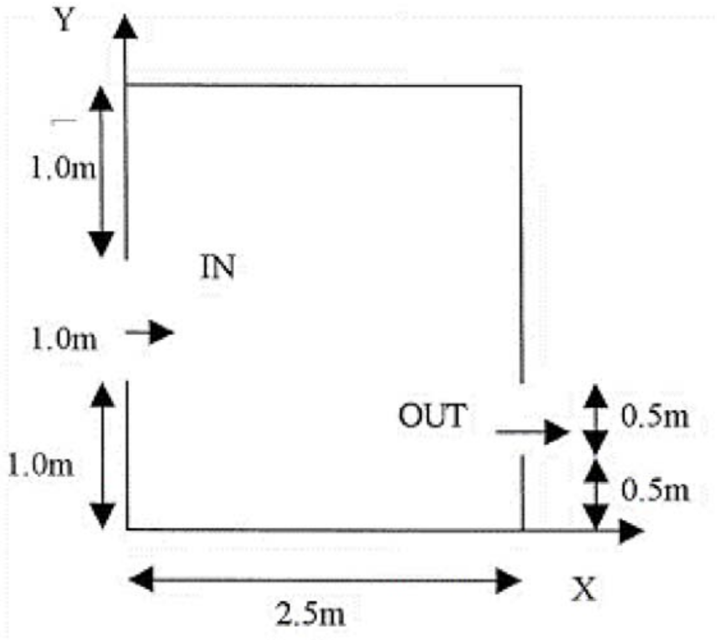
Problem statement
A 'butterfly effect' inside Buildings?



Butterfly effect :
Extreme sensitivity leads to an unpredictable system

How sensitive is the airflow in a ventilated room for very small parameter changes?

Numerical case study



$$\frac{\partial u}{\partial t} = -\frac{\partial (uu)}{\partial x} - \frac{\partial (vu)}{\partial y} - \frac{\partial p}{\partial x} + \frac{1}{\mathrm{Re}}\nabla^2 u$$

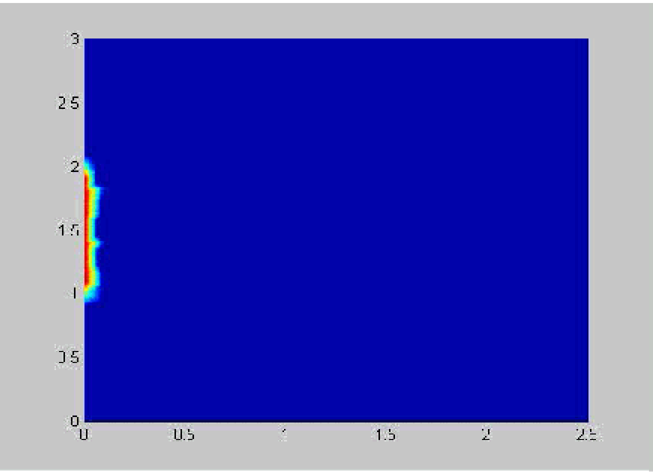
$$\frac{\partial v}{\partial t} = -\frac{\partial (uv)}{\partial x} - \frac{\partial (vv)}{\partial y} - \frac{\partial p}{\partial y} + \frac{1}{\mathrm{Re}}\nabla^2 v + \frac{Gr}{\mathrm{Re}^2}T$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

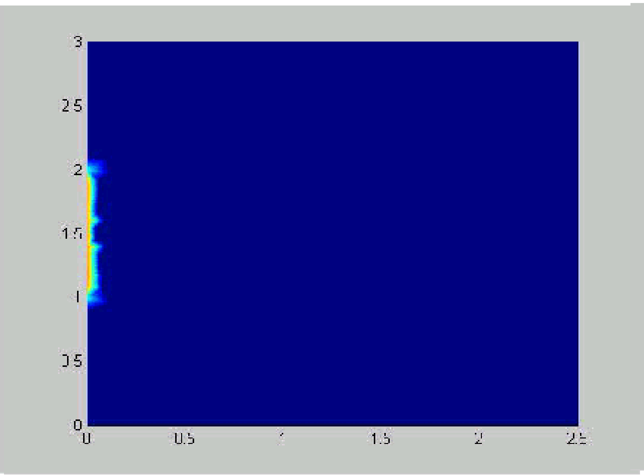
$$\frac{\partial T}{\partial t} = -\frac{\partial (uT)}{\partial x} - \frac{\partial (vT)}{\partial y} + \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\nabla^2 T$$

Sinha et al. 2000 Energy and Buildings 32, pp121-129

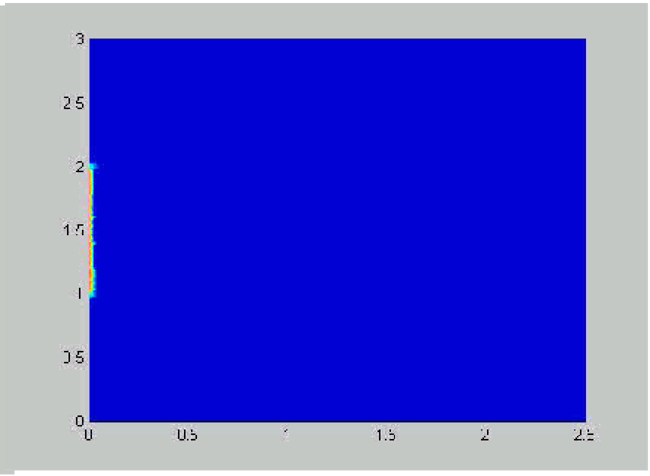
Simulation using Comsol



Re = 50; Gr =0



Re = 1000; Gr =0

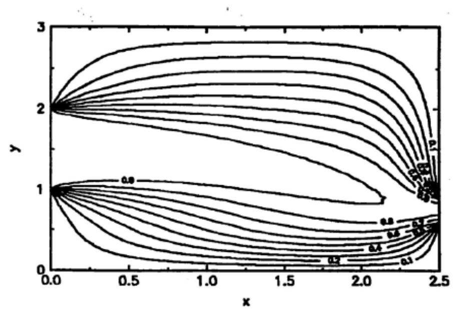


Re = 1000; Gr =~10^7

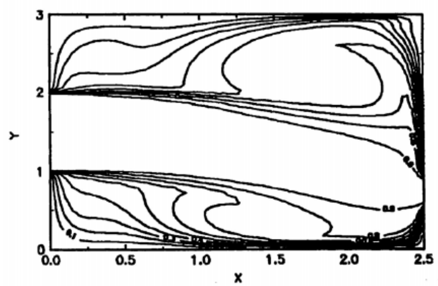
TU/e Technische Universiteit **Eindhoven** University of Technology

Verification

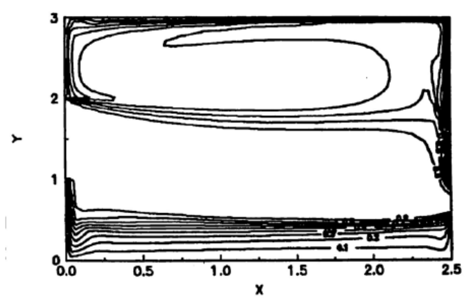
Sinha et al. 2000



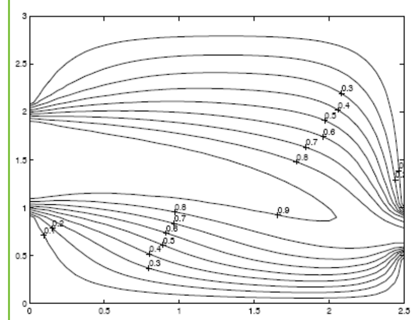
a) Re = 50 , Gr=0

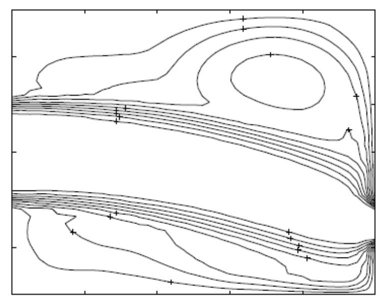


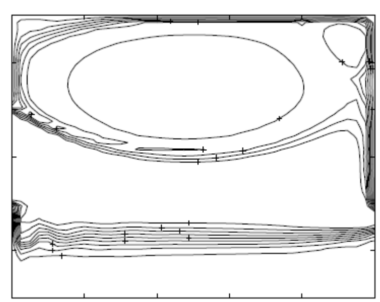
b) Re =1000 , Gr =0



c) Re = 1000 , Gr =2.5 10 ^{7}







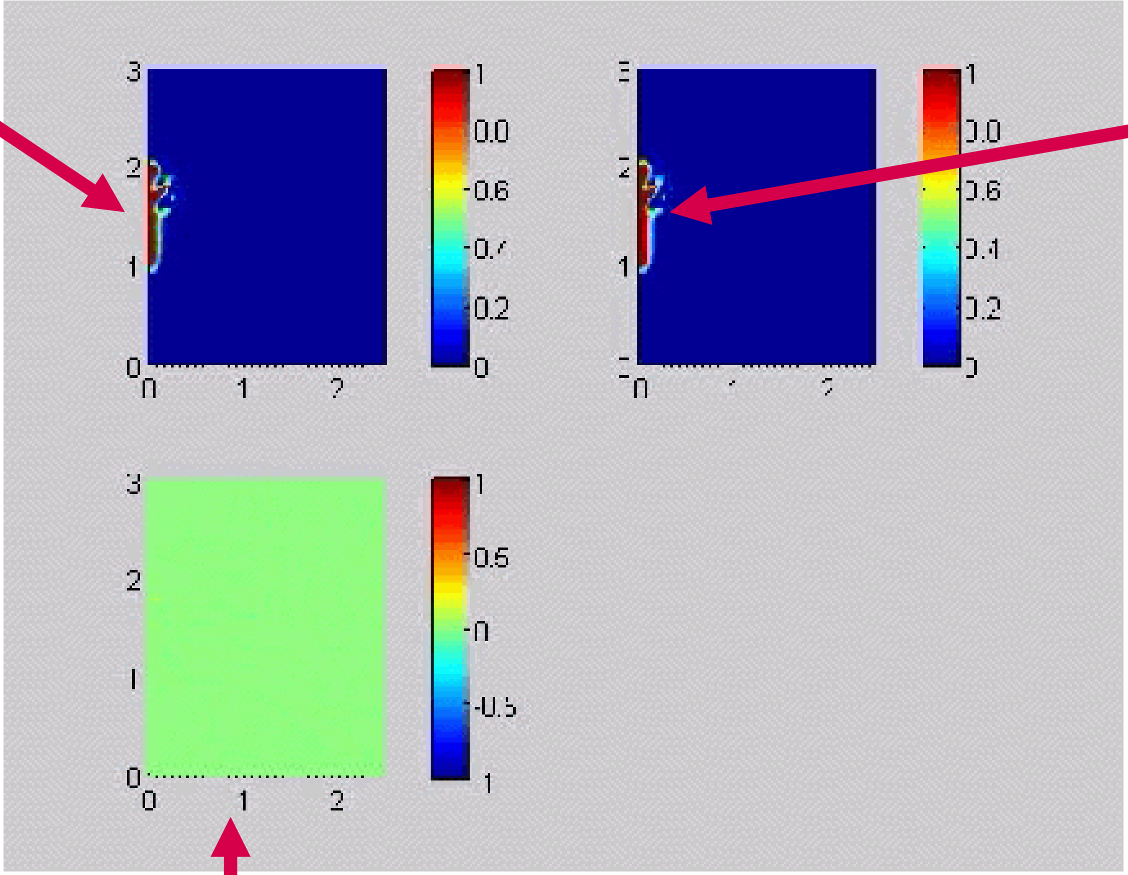
Comsol

Technische Universiteit Eindhoven University of Technology

Air supply sensitivity

Air supply = 1

Air supply = 0.98



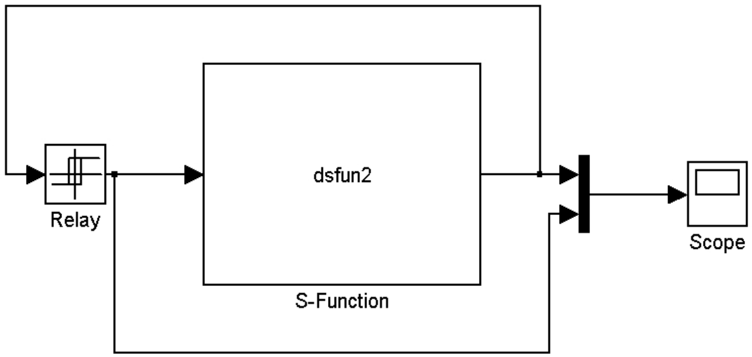
Difference between top figures

Technische Universiteit
Eindhoven
University of Technology

Switching model Comsol/SimuLink

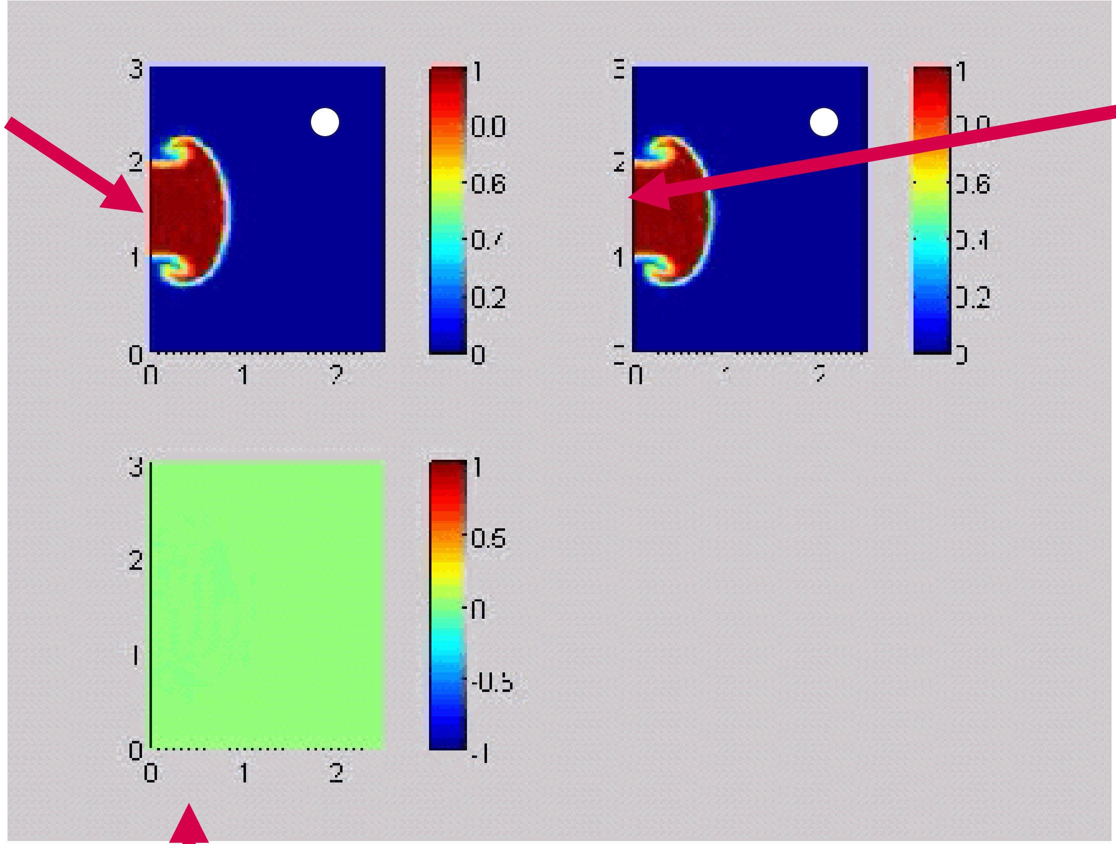
- **Using SimuLink & S-Functions**

(Schijndel, A.W.M. van, 2005, Implementation of FemLab in S-Functions, 1ST FemLab Conference Frankfurt, pp324-329)



Switching sensitivity without buoyancy

Switching:
<0.30 hot air
>0.50 cold air

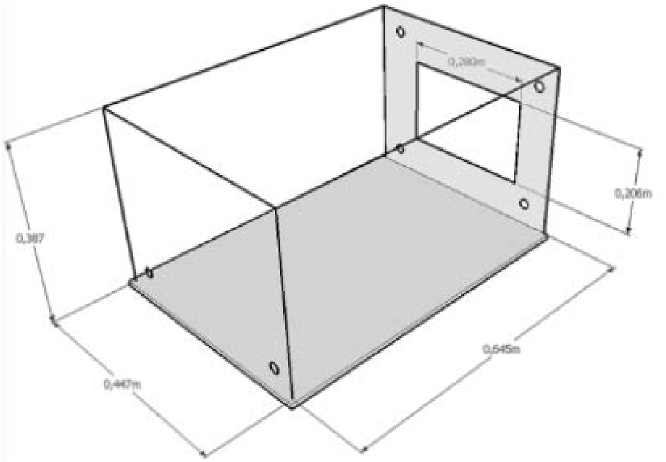


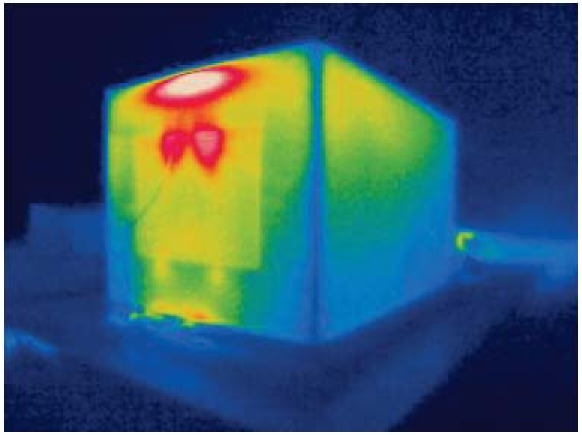
Switching:
<0.32 hot air
>0.48 cold air

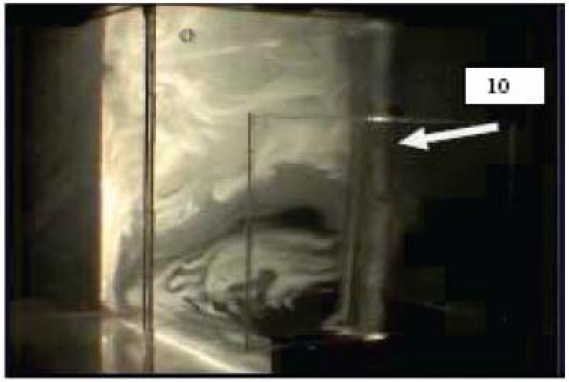
Difference between top figures

TU/e Technische Universiteit Eindhoven University of Technology

Experimental case study: a scale model







Technische Universiteit
Eindhoven
University of Technology

Simulation of the scale model

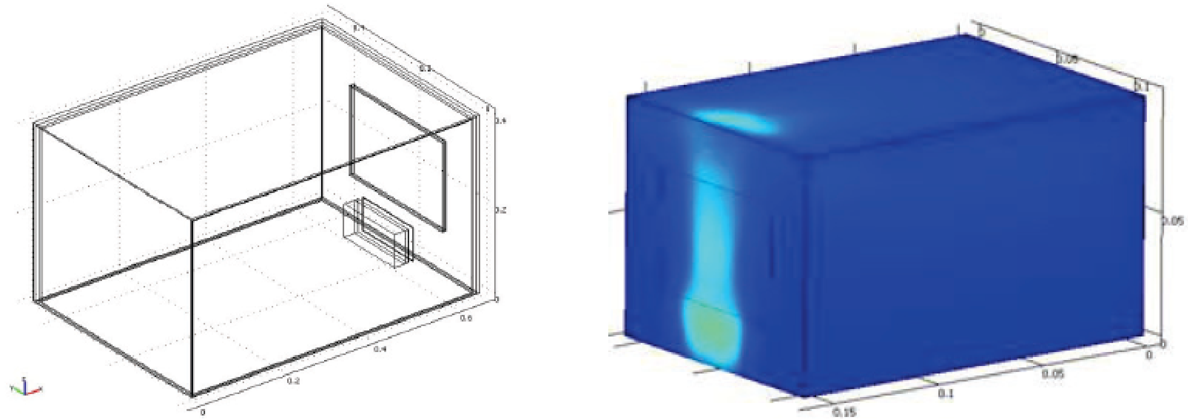


FIG 9. Left: Right: Simulated surface temperature

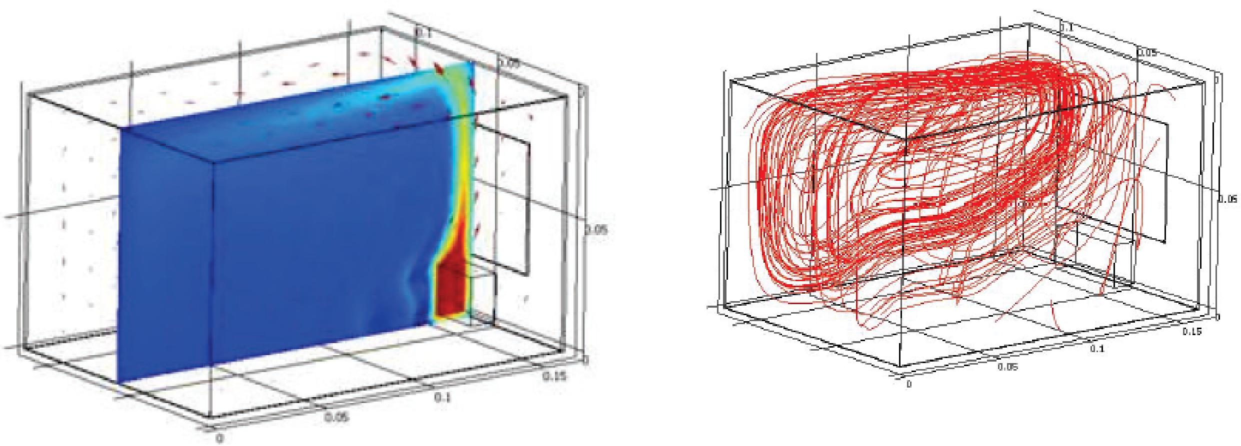


FIG 10. Left: The temperature and velocity after 900 seconds; Right: The air circulation

Technische Universiteit
Eindhoven
University of Technology

Conclusions numerical work

2D Simulation with buoyancy

- Chaotic behavior is already observed by changing the supply air temperature from 22 °C into 21.9 °C.

2D Simulation without buoyancy & switching

- Minor chaotic behavior is observed by a small change in the air supply control parameters

Future research

- Simulation with buoyancy with switching

Conclusions experimental work

Scale model with buoyancy

- Chaotic behavior still under investigation.

Scale model with buoyancy & switching

- Future research, chaotic behavior is expected

Question

- What does this mean for the predictability of a full scale indoor climate?

Thank you !