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Preface

This book grew out of notes used in a modeling course attended by upper-division undergraduate mathematics, science, engineering, and economics students that I and others have taught over several years in the mathematics department at Wright State University. From the beginning I had to rely heavily on home-made lecture notes, as I could not find a text suitable for beginners, and yet dealt with models challenging to a third or fourth year student. Same books had good modeling examples, but the students didn't like to read them as they didn't develop the mathematical structure; however, I found the "easy-reading", generalized, modeling texts too light on material. After a couple of years of hearing from colleagues that they would like the same sort of text that I envisioned, I started to write this book.

The technical prerequisites for a reading of the text are minimal: a solid calculus course, some exposure to differential equations (such as is sometimes found in calculus courses) , and a little matrix algebra. Although the use of the whole book would be ideal for a one-year introduction to applied math, the form of the book reflects to some extent a couple of one-quarter courses that are taught at Wright State University: MTH 306/606 Mathematical Modeling, attended by junior-level and senior-level mathematics majors and graduate students in math education, biomedical engineering, biophysics, economics, mechanical engineering, electrical engineering, computer science, and statistics; and MTH 333/533 Partial Differential Equations, attended by junior-level and senior-level math and physics majors, as well as graduate students in electrical, mechanical, and biomechanical engineering. A student can take the courses in either order. The modeling course covers, typically, Chapters 2 through 6 with supplementary material from Chapters 7, 8, and 9. The PDE course is based on Chapters 9,10, and 11, with supplementary material from chapters 3, and 5,6.

At the same time, I have tried to make it as suitable for self-study I as I possibly could. Besides making the subject accessible to the large number of students who find themselves without a relevant course, this should be of help to a teacher who wants to devote a large amount of classtime to the discussion of projects. The wide range of examples and mathematical techniques will broaden the student's vista and help get across the idea that there is no fixed set of tools for modeling.

In fact, the chapters are largely self-contained, although there are definite connections and built-in redundancies so that the student can see the same idea used in a different context. In this way, a clear

stream can be followed (see the figure at the end of this preface). As the graphic indicates, Chapter 3 is the foundation of the book. It is natural to proceed from there to Chapter 4. One can go directly to Chapter 6 if probabilistic methods are to be the main content of the course. Chapters 1 and 2 have found use in surveying and exploring with students some of the simple ideas of applied mathematics, but their quick pace may dismay the insecure student. In particular, the first half of Chapter 1 is meant as entertainment; if the student doesn't find it so, it could be skipped. The rest of chapter 1 contains some background material. The chapters themselves are written in a sort of newspaper pyramid style so that one can either study a chapter thoroughly or simply read the first part of each chapter. Sections that are not necessary for later chapters and/or require more mathematical sophistication and/or ask for much classtime are marked with an asterisk (*).

The pedagogical intent of this book is to help develop in students a feeling for the use of mathematics as a tool in the understanding of the world. A common complaint of students when beginning the modeling course is that they "lack the physical intuition" to be able to model. The book is put together with the feeling that a "modeling intution" can be nurtured in the mathematics student. It is mainly a matter of developing confidence, not just in problem solving, but in ability to approach complicated phenomena by asking a few simple questions. To this end and to encourage students to do much of the thinking on their own, exercises are built into the narrative. These exercises function as a governor: if they are trivial, the student can pick up speed; if they are not quite understood, a rereading of the text will be called for. Some require little work and may function simply to keep the student's pencil sharpened, but are designed to help the student take first responsibility for learning. Other exercises require some amount of thinking and/or search for data. There are also problem sections at the end of the chapters; these consist mainly of particular models, some of which may be suitable for a class project. Several independent trails can be followed through the problem sections. For example, chemical reactions and compartment models are introduced in the problems of Chapter 4 and reappear in problem sections in several later chapters.

It is clear to its teachers that modeling is not mathematics per se, but certainly the point of it is to use mathematics to show the underlying links between apparently disparate phenomena. Indeed, in many mathematics books, one often comes across a footnote remarking that the subject presently under discussion can also be clothed a different way, and in such and such a context. In this book these footnotes have been collected and expanded. I'd like to make the seemingly paradoxical statement that an engineer or scientist will want to take a modeling course, not to learn abstractions, but rather the opposite: to become better acquainted with concrete phenomena. Often in engineering texts one sees a formula derived and then some magical mathematics applied to it, and the result is a "theoretical rule of thumb" that appears in a box on the page. The student engineer accepts the boxed result as a substitution for the phenomenon and while becoming a practicing engineer will continue to do so. If he or she the engineer ends up doing something more than paper work, he or she will notice a disparity and the boxed formula is thrown into the trash bin of "theory" which is disparagingly regarded as being unrelated to the "real world". On the other hand, the development of mathematical skills is necessary for a development of modeling skills. It may turn out that the mathematical technique needed for a particular model is yet to be found. Many problems started out as modeling problems and turned into areas of (pure) mathematics. Some of the great mathematicians spent an extraordinary amount of their time on modeling (Archimedes, Newton, Euler, Bernoulli, and others).

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