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Preface

Creation of this book was one of two related activities undertaken by a committee of colleagues to recognize Clayton Aucoin on his 65th birthday for his contributions to mathematical sciences education. A book on modeling seemed an appropriate tribute to one who insisted that modeling play a central role in the mathematical sciences curriculum. That viewpoint guided the development of the Clemson graduate program in the mathematical sciences, recognized as an exemplar of a successful program. Without doubt, it was a model-oriented multidisciplinary approach to the curriculum that earned this recognition. The second celebratory activity was the creation of the Clayton and Claire Aucoin Scholarship Fund at Clemson University. All royalties generated by sales of this volume will help support students in the mathematical sciences at Clemson University.

To create this modeling book, a five-person Editorial Committee was formed from department members whose areas of expertise span the mathematical sciences: applied analysis, computational mathematics, discrete mathematics, operations research, probability, and statistics. The Committee believes that modeling is an art form and the practice of modeling is best learned by students who, armed with fundamental methodologies, are exposed to a wide variety of modeling experiences. Ideally, this experience could be obtained through a consultative relation in which a team works on actual modeling problems and their results are subsequently applied. But such an arrangement is often difficult to achieve, given the time constraints of an academic program. This modeling volume therefore offers an alternative approach in which students can read about a certain model, solve problems related to the model or the methodologies employed, extend results through projects, and make presentations to their peers. Consequently, this volume provides a collection of models illustrating the power and richness of the mathematical sciences in providing insight into the operation of important real world systems.

Indeed, recent years have witnessed a dramatic increase in activity and urgency in restructuring mathematics education at the school and undergraduate levels. One manifestation of these efforts has been the introduction of mathematical models into early mathematics education. Not only do such models provide tangible evidence of the utility of mathematics, but the modeling process also invites the active participation of students, especially in the translation of observable phenomena into the language of mathematics. Consequently, it is not surprising that one can now find over fifty textbooks dealing with mathematical models or the modeling process.

Reform of graduate education in the mathematical sciences is no less important, and here too mathematical modeling plays an important role in suggested models of curricular restructuring. Whereas there are several excellent textbooks that provide an introduction to mathematical modeling for undergraduates, there are few books of sufficient breadth that focus on modeling at the advanced undergraduate or beginning graduate level. The intent of this book is to fill that void.

The volume is conceptually organized into two parts. Part I, comprising three chapters written by well-known experienced modelers, gives an overview of mathematical modeling and highlights the potentials (as well as pitfalls) of modeling in practice. Chapter 1 discusses the general components of the modeling process and makes a strong case for the importance of modeling in a modern mathematical sciences curriculum. Chapter 2, although not intended as a portmanteau of specific techniques, contains important ideas of a general nature on approaches to model building. It uses simple models of physics and more realistic models of efficient economic systems to drive home its points. Chapter 3 describes an experienced modeler's decade-long "odyssey" of modeling the AIDS epidemic. As in most journeys, the traveler often encounters new information as the trip evolves. Initial plans must be modified to meet changing conditions and to use new information in an intelligent manner as it becomes available. Road maps that were current at the beginning of the journey may not show the detours ahead. So too does this chapter illustrate the evolutionary nature of successful model building.

Part II is a compendium of sixteen papers, each a self-contained ex-position on a specific model, complete with examples, exercises, and projects. Diverse subject matter and the breadth of methodologies employed reinforce the flexibility and power of the mathematical sciences. To avoid the appearance that one "correct" model has been formulated and analyzed, the treatment of most modeling situations in Part II is deliberately left open ended. A unique feature of many of these models is a reliance on more than one of the synergistic areas of the mathematical sciences. This multidisciplinary approach justifies use of that word in the book's title.

The level of presentation has been carefully chosen to make the material easily accessible to students with a solid undergraduate background in the mathematical sciences. Specific prerequisites are listed at the start of each chapter appearing in Part II. Included with each model is a set of exercises pertaining to the model as well as projects for modification and/or extension of results. The projects in particular are highly appropriate for group activities, making use of the reinforcing contributions of group members in a collaborative learning environment. A number of the chapters discuss computational aspects of implementing the studied model and suggest methods for carrying out requisite calculations using high-level, and widely available, computational packages as Maple, Mathematica, and MATLAB).

This book may be viewed as a handbook of in-depth case studies that span the mathematical sciences, building upon a modest mathematical background (differential equations, discrete mathematics, linear algebra, numerical analysis, optimization, probability, and statistics). It makes the book suitable as a text in a course dedicated to modeling, in which students present the results of their efforts to a peer group. Alternatively, the models in this volume could be used as supplementary material in a more traditional methodology course to illustrate applications of that methodology and to point out the diversity of tools needed to analyze a given model. In either situation, since communication skills are so important for successful application of model results, it is recommended that students, working alone or in groups, read about a specific model, work the exercises, modify or extend the model along the suggested lines, and then present the results to the rest of the class. This volume will also be useful as a source book for models in other technical disciplines, particularly in many fields of engineering. It is believed that readers in other applied disciplines will benefit from seeing how various mathematical modeling philosophies and techniques can be brought to hear on problems arising in their disciplines. The models in this volume address real world situations studied in chemistry, physics, demography, economics, civil engineering, environmental engineering, industrial engineering, telecommunications, and other areas.

The multidisciplinary nature of this book is evident in the various disciplinary tools used and the wealth of application areas. Moreover, both continuous and discrete models are illustrated, as well as both stochastic and deterministic models. To provide readers with some initial road maps to chart their course through this volume, several tables are included in this preface. In keeping with the multidimensional nature of the models presented here, the chapters of Part II are listed in simple alphabetical order by the author's last name. Whereas in most mathematics texts, one must master the concepts of early chapters to prepare for subsequent material, this is clearly not the case here. One may start in Chapter 5 if cryptology catches your fancy or in Chapter 12 if bursty traffic behavior is your cup of tea.

Disciplinary Tools	Chapters
applied analysis	11,12
data analysis	3,16,19
data structures	13
differential equatiolls	2,3,4,7,9,11,12,17,18.19
dynamical systems	4,11
graph theory	13,15
linear algebra	2,4,5,11,14,17
mathematical programming	2,10,16
modern algebra	6,13,14

number theory	5,6
numerical analysis	9,17
probability	8,10,12,13
queueing theory	8,12
scientific computing	9.17.19
statistics	8,10

Application Areas	Chapters
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demography	3,4
economics	2,16
environmental engineering	9,16
error-correcting codes	14
manufacturing	10,14
physics	7,11,17,18
public health	3,4
queueing systems	8,12

Model Types	Chapters
continuous	2,3,4,7,8,9,11,12,17,18,19
discrete	4,5,6,10,13,14,15,16
deterministic	2,3,4,5,6,7,9,10,11,14,15,16,17,18,19
stochastic	3,4,8,10,12,13

The book concludes with an appendix that provides an overview of the evolution and structure of graduate programs at Clemson University, programs that rely heavily on the pedagogical use of mathematical modeling. It recapitulates the fundamental importance of mathematical modeling as a driving force in curriculum reform, echoing the points made in Chapter 1 and concretely illustrating the need for a multidisciplinary approach.

The Editorial Committee has done its best to provide a sample of the wide range of modeling techniques and application areas. This book will be considered a success if it has whetted the reader's appetite for further study. Consequently, references to both printed materials and to websites are provided within the individual chapters. Supplementary material related to the models developed in this volume can be found at the website <http://www.math.clemson.edu/modeling/> for this book.

Acknowledgments

We are indebted to Dawn M. Rose for her unflagging and careful efforts in editing this unique volume.

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