# PREFACE

Controversy has persisted in recent years over how to teach mathematical modelling. There are two extreme schools of thought. One believes that a modelling course should have little structure and be dominated by "case studies," open-ended exercises requiring data collection and research. These might lead students down a great many unforeseen avenues of creativity but might also lead them nowhere. The other school believes that a modelling course should be highly structured and dominated by what, for want of a better phrase, I will refer to as "modelling exercises." These are exercises that involve formulation and interpretation but bear a more certain relationship to the principal content of the course. Between these two extremes, of course, lie many compromises, one of which forms the basis for this textbook.

Before describing it, however, let me argue that there is an important sense in which the second school of thought should take precedence over the first. Most college majors in mathematics do not become professional mathematicians. Rather, they join the graduate workforce as businesspeople, as financial planners, as politicians, or in a number of other careers, and it is far more important that they can understand, interpret, criticize, and appreciate the mathematical models of others than that they are able to develop their ow'n models. Just as drivers need not build and repair their cars but should know how they work, so consumers of mathematical models need not produce them but should understand both their usefulness and their limitations. Naturally, it remains desirable that math majors be able to develop their own models, but this is essential only to the few who subsequently make model development their career. On the other hand, the capacity to understand, interpret, criticize, and appreciate models is essential to all.

With this in mind, 1 have written this text for the deserving many, not simply a chosen few. Yet I have not neglected the specific needs of those who aspire to building their own models, although I have not explicitly addressed them until relatively late in the book. This is deliberate. Between the two extreme schools of thought this book has adopted a dynamic compromise. in which it begins by espousing the second extreme and gradually evolves toward the first. This approach enables the book to emphasize both the validation of mathematical models and the rationale behind improving them. Each of these features is novel, as will be clear to anyone familiar with Murthy and Rodin's (1987) review of books on mathematical modelling.

This is a book about the process of modelling. Its approach is heuristic but systematic and embodies my belief that the three most fundamental ideas in mathematical modelling are transience, permanence, and optimality. Because models incorporating these ideas may adopt either a deterministic or a probabilistic viewpoint, there are six combinations of idea and viewpoint. These define the structure of Chapters 1-3 and Chapters 5-7 of the text, which proceed in parallel as depicted in the following diagram.

| Idea / View | Transience | Permanence | Optimality |
|---|---|---|---|
| Deterministic | Chapter 1<br>Growth and Decay-<br>Dynamical Systems | Chapter 2<br>Equilibrium | Chapter 3<br>Optimal Control<br>and Utility |
| Probabilistic | Chapter 5<br>Birth and Death.<br>Probabilistic Systems | Chapter 6<br>Stationary<br>Distributions | Chapter 7<br>Optimal Decision<br>and Reward |

In applying these ideas, mathematical modellers are called upon to play conflicting roles. On the one hand, they must be creative anists, assuming boldly perhaps what no one has assumed before. On the other hand, they must be critics, scrupulously doubting whether their models provide an adequate description of reality. In practice, of course, the distinctions I have drawn between ideas, roles, and even viewpoints are artificial; but it greatly benefits a beginning student of mathematical modelling to focus on a single aspect at a time, even though the other aspects cannot be entirely ignored. Accordingly, the critical aspect of modelling is the subject of Chapter 4, its creative aspect the subject of Chapter 9, and the interface between the two the subject of Chapter 8. The remainder of the book, Chapters 10-12, is a reinforcement of the ideas developed in the first nine chapters.

The goal of this book is to let readers acquire both critical and creative modelling skills and the confidence to use them. All other matters are regarded as secondary. Thus, as far as is possible, technical mathematical details are explored in exercises, or occasionally in footnotes, rather than in the main body of text. To distinguish such purely technical exercises from those that are more directly related to the goals of the book, modelling exercises are denoted by a single asterisk and case studies by a double asterisk. But not every detail has found its way into a footnote or exercise, and the book cannot be used profitably unless pencil and paper at all times accompany the reader. The book is to be studied, not simply read.

The book's goal is pursued through a layered approach, with frequent revisitations to earlier sections, so that even the most sophisticated models are perceived as merely natural outgrowths of less sophisticated ones. As proclaimed in the title, this layered approach is unashamedly concrete. Philosophical points are not discussed until several illustratory examples have already been introduced. Even then the discussion is brief, as at the beginning of Chapter 4, or in Chapter 9, where the art of adapting, extending, and combining is first discussed formally, even though it characterizes every model developed in the text. At all times, I have tried to avoid unnecessary abstractions. I have taken particular care never to introduce a utility function as $F(x)$, with constraints on the signs of the derivatives of $F$; instead, I give the dependence of $F$ on $x$ explicitly, so that properties of the derivatives are self-evident. The loss of generality is more than compensated by the gain in comprehension. Indeed my experience suggests that the educational value of a modelling course to the average student is decided by this factor more than by any other.

This book is primarily intended for a senior level course that gives equal weight to deterministic and probabilistic modelling. Accordingly, minimal mathematical prerequisites for mastery of its entire contents are the standard calculus sequence (including the Newton-Raphson method) and first courses in linear algebra, ordinary differential equations, and probability and statistics. Probability and statistics are reviewed in Appendix 1. A few sections require access to computer packages for solving linear programs or ordinary differential equations (now almost universally available), while some knowledge of numerical analysis is desirable (though by no means necessary). A modelling course need not include the book's entire contents, however, and alternative uses are described in an accompanying instructor's manual. In particular, there is ample material in Chapters 1-4 and Chapters 8-11 for a course that requires no probability and statistics.

But this book is designed to be also suitable for study alone, perhaps by beginning graduate students or by professionals whose background in mathematical modelling is weak. For this reason, if an exercise is crucial to later developments and of even moderate difficulty, then its solution (or a possible solution) appears at the back of the book. I therefore assume that my readers are mature enough not to consult a solution until a serious attempt has been made at the problem. Solutions to most other exercises appear in the instructor's manual.

By using plurals and genderless singulars, I have thus far avoided the vexed question of whether my reader is male or female. The lack of epicene pronouns and possessive adjectives in the English language is regrettable, and I have long advocated the use of "their" to mean "his or her." But grammarians may be offended by this, and the continual use of "his or her" is deplorably inelegant. I therefore felt that odd-numbered chapters should have one gender, even-numbered chapters another, and the toss of a coin decided that males would be odd, females even. Thus Chapters 1, 3, 5, 7, 9, and 11 are male, and Chapters 2, 4, 6, 8, 10, and 12 are female. This convention is not without minor inconveniences - for example, the forester we meet in Chapter 3 has to change his sex before she enters Chapter 4 - but this is surely preferable to the alternatives.

The reviews of the first edition of this book have all been very positive and pleasing. I haven't fixed what isn't broken. Nevertheless, for this revised edition I have corrected all known errors, I have added a few exercises, and I have updated both the list of references and other time-sensitive material.

Finally, this is a substantially original book. The models are based on the scientific literature, specifically, on the sources identified in Appendix 2. But I have made it a point of honor to personalize each model by deriving it from scratch, starting with a blank piece of paper and working through all the details myself. This has led not only to differences of emphasis and approach but also to new extensions and variations. Moreover, not only are models freely adapted, they are presented here in a fresh perspective, as the interlocking fragments of a mosaic. This is my mosaic of the modelling process, carefully assembled from my teaching experience, in which every fragment contributes to the synthesis of a coherent picture. What the picture shows me is that even a modest amount of mathematics - no more than should be expected of every college major in business, science, or engineering - is enough to describe a wide variety of phenomena, offer penetrating insights, and contribute effectively to rational decision making. If that is also what the picture shows my readers, then I shall be satisfied with what I have written.

Acknowledgement

I am deeply indebted to the many scientists whose mathematical models have formed a basis for this textbook. I wish to express my thanks to all of them, and to each in a measure proportional to my indebtedness.

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Tallahassee, Florida MICHAEL MESTERTON-GIBBONS

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