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## Preface

The Mathematical Association of America's Committee on the Undergraduate Program in Mathematics (CUPM) has long recommended that "Students should have an opportunity to undertake 'real world' mathematical modeling projects, either as term projects in an operations research course, as independent study, or as an internship in industry"(1). That report goes on to add that a modelling experience should be included within the common core of all mathematical sciences majors. Further, this experience in modeling should start early: "... to begin the modeling experience as early as possible in the student's career and reinforce modeling over the entire period of sludy."(2)

To facilitate an early initiation of the modeling experience, the first edition of this text was designed to be taught concurrently or immediately after an introductory business or engineering calculus course. In this edition, we have added chapters treating discrete dynamical systems, linear programming and numerical search methods, and an introduction to probabilistic modeling. Additionally, we have expanded our introduction of simulation. The additional topics in discrete mathematics now make it possible to organize an entire course that does not require calculus. Chapters that require a concurrent introductory business or engineering calculus course are Chapter 8, Continuous Optimization Models; Chapter 10, Modeling with a Differential Equation; and Chapter 11, Modeling with Systems of Differential Equations. We have organized the text so that the course may begin in the first semester of freshman year. Chapter 1, Graphs of Functions as Models, requires only the notion of how the signs of the derivatives determine the shape of the graph of a function. The chapter can be skipped if desired. Chapter 5 uses calculus to derive the normal equations for the least-squares criterion of "best fit." The proof may be skipped if desired. Otherwise, the first seven chapters of the text do not require prior or concurrent experience in calculus. We now describe how our book is designed for a first course in modeling.

(1) Mathematical Association of America, Committee on the Undergraduate Program in Mathematics. Recommendations for a General Mathematical Sciences Program (Washington, DC: Mathematical Association of America, 1981). p. 13.

(2) ibid., p. 77.

### Goals and Orientation

The course is a bridge between the study of mathematics and the applications of mathematics to various fields, and it is a transition to the significant modeling experiences recommended by the CUPM. The course affords the student an early opportunity to see how the pieces of an applied problem fit together. The student investigates meaningful and practical problems chosen from common experiences encompassing many academic disciplines, including the mathematical sciences, operations research, engineering, and the management and life sciences.

This text provides an introduction to the entire modeling process. The student will have occasions to practice the following facets of modeling:

1. Creative and Empirical Model Construction: Given a real-world scenario, the student learns to identify a problem, make assumptions and collect data, propose a model, test the assumptions, refine the model as necessary, fit the model to data if appropriate, and analyze the underlying mathematical structure of the model to appraise the sensitivity of the conclusions when the assumptions are not precisely met.

2. Model Analysis: Given a model, the student learns to work backward to uncover the implicit underlying assumptions, assess critically how well those assumptions fit the scenario at hand, and estimate the sensitivity of the conclusions when the assumptions are not precisely met.

3. Model Research: The student investigates a specific area to gain a deeper understanding of some behavior and learns to use what has already been created or discovered.

We have designed the text to enhance a student's problem-solving capabilities. For purposes of discussion we identify the following steps of the problem-solving process:

1. Problem identification

2. Model construction or selection

3. Identification and collection of data

4. Model validation

5. Calculation of solutions to the model

6. Model implementation and maintenance

In many instances the undergraduate mathematical experience consists almost entirely of doing step 5: calculating solutions to models that are given. There is relatively little experience with "word problems", and what there is deals mainly with problems that are short (to accommodate a full syllabus) and often contrived. Such problems require the student to apply the mathematical technique currently being studied, from which solution to the model is calculated with great precision. For lack of experience, consequently, students often feel anxious when presented with a scenario for which the model is not given or for which there is no unique solution, and then are told to identify a problem and construct a model addressing the problem "reasonably well".

With this in mind, we feel that in an introductory modeling course students should spend a significant amount of time on the first several steps of the process just described-learning how to identify problems, construct or select models, and figure out what data need to be collected-progressing from relatively easy scenarios to more difficult ones. It is probably unreasonable to expect an average student to excel in a semester-long project on the first attempt. It takes time and experience to develop skill and confidence in the modeling process. We have found that involving students in the mathematical modeling process as early as possible, beginning with short projects, facilitates their progressive development and confidence in mathematics and modeling.

Many modeling texts present "type models" such as various inventory models for determining optimal inventory strategies. Students then learn to select an appropriate model for a particular situation. This approach has merit, and model selection is a valid step in the problem-solving process. However, undergraduate students often do not comprehend the assumptions behind a model, and only rarely do they take into consideration the appropriateness and sensitivity of those assumptions. Therefore, we emphasize model construction to promote student creativity and to demonstrate the artistic nature of model building, including the ideas of experimentation and simulation. Although we do discuss fitting data to chosen model types, our concentration is still on the entire model-building process, leaving the study of type models for more advanced courses.

## Student Background and Course Content

Because our desire is to initiate the modeling experience as early as possible in the student's program, the only prerequisite for Chapters 8, 10, and 11 is a basic understanding of single-variable differential and integral calculus. Although some unfamiliar mathematical ideas are taught as part of the modeling process, the em- phasis is on using mathematics already known by the students after completing high school. The modeling course will then motivate students to study the more advanced courses such as linear algebra, differential equations, optimization and linear programming, numerical analysis, probability, and statistics. The power and utility of these subjects are intimated throughout the text.

Although there are strong arguments to include such courses as advanced calculus, linear algebra, differential equations, probability, and statistics as prerequisites to an introductory modeling course, such a requirement necessitates postponing the course until the junior or senior undergraduate year, delaying the student's exposure to real-world applications. It also cuts off a number of student beneficiaries (namely, those non-mathematics majors who cannot satisfy all the prerequisites). Though our philosophy differs somewhat, this text still serves the more advanced student who has taken more mathematics courses. Certain sections of the text can be covered more rapidly by the advanced student, allowing more time for deeper extensions of the material as suggested by the projects for each chapter.

Further, the scenarios and problems in the text are not designed for the application of a particular mathematical technique. Instead, they demand thoughtful ingenuity in using fundamental concepts to find reasonable solutions to "open-ended" problems. Certain mathematical techniques (such as dimensional analysis, curve fitting, and Monte Carlo simulation) are presented because often they are not formally covered at the undergraduate level. Instructors should find greal flexibility in adapting the text to meet the particular needs of students through the problem assignments and student projects. We have used this material to teach courses to both undergraduate and graduate students, and even as a basis for faculty seminars.

## Organization of the text

The organization of the text is best understood with the aid of Figure 1. The first five chapters are directed toward creative model construction and an overview of the entire modeling process. We begin with the construction of graphical models, which provides us with some concrete models to support our discussion of the modeling process in Chapter 2. This approach also provides a transition into model construction by first involving the student in model analysis. Next we classify models and analyze the modeling process. At this point students can really begin to analyze scenarios, identify problems, and determine the underlying assumptions and principal variables of interest in a problem. This work is preliminary to the models they will create beginning in Chapter 3. (The order of Chapters 1 and 2 may be reversed, although we have found the current order best for capturing student interest and reducing student anxiety.) In their first modeling experience. students are quite anxious about their creative abilities and how they are going to be evaluated. For these reasons wehave found it advantageous to start them out on familiar ground by appealing to their understanding of graphs of functions and having them learn model analysis. The book blends mathematical modeling techniques with the more creative aspects of modeling for variety and confidence building, and gradually the transition is made to the more difficult creative aspects.

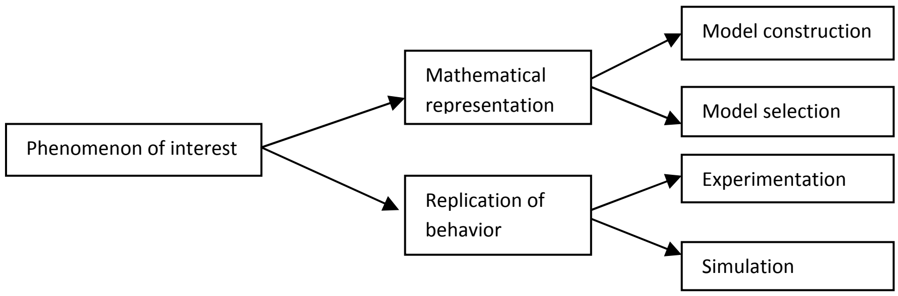


FIGURE 1 The organization of the text follows the above classification of the various models

In Chapter 3 we present the modeling of "change" using difference equations.  Students are provided data from situations they have encountered in daily life.  They are given the opportunity to construct and test models for various situations, beginning with scenarios that can be modeled exactly, before approximating more difficult scenarios. In Chapter 4 we present the concepts of proportionality and geometric similarity and use them to construct mathematical models for some of the previously identified scenarios. Students formulate tentative models or submodels and begin to learn how to test the appropriateness of the assumptions. In Chapter 5 model fitting is discussed, and in the process several optimization models are developed and are revisited in Chapters 8 and 9.

Chapters 6 and 7 are dedicated to empirical model construction. Chapter 6 begins with fitting simple one-term models to collected sets of data and progresses to more sophisticated interpolating models, including polynomial smoothing models and cubic splines. Simulation models are discussed in Chapter 7. An empirical model is fit to some collected data, and then Monte Carlo simulation is used to duplicate the behavior being investigated. The presentation motivates the eventual study of probability and statistics.

Chapters 8 and 9 are devoted to the study of optimization. At this point in the course students have encountered many situations that ask them to find the best solution. We begin Chapter 8 by allowing students to formulate optimization models of various types. We then classify the models by their mathematical structures. In Chapter 8 students get the opportunity to solve continuous optimization problems requiring only the application of elementary calculus. An introduction to constrained optimization problems is provided as well. In Chapter 9, linear programming and several numerical search methods are presented. The treatment of linear programming includes analytical as well as graphical methodologies. An introduction to the important topic of sensitivity analysis is provided. The chapter concludes with an introduction to numerical search methods including the dichotomous and golden section methods.

In Chapters 10 and 11 dynamic (time varying) scenarios are treated. These chapters build on the discrete analysis presented in Chapter 3 by now considering situations where time is varying continuously. We begin by modeling initial value problems in Chapter 10 and progress to interactive systems in Chapter 11, with students performing a graphical stability analysis. Students with a good background in differential equations can pursue analytical and numerical stability analyses as well, or they can investigate the use of difference equations or numerical solutions to differential equations by completing the projects.

Chapter 12 provides an introduction to probabilistic modeling. The topics of Markov processes, reliability, and linear regression are introduced, building on scenarios and analysis presented previously in the course. The text concludes with Chapter 13, which is devoted to dimensional analysis, a topic of great importance in the physical sciences and engineering because it represents a means of significantly reducing the experimental effort required when constructing models based on data collection. We also include an introduction to the construction of models of similitude.

The text is arranged in the order we prefer in teaching our modeling course. However, the order of presentation may be varied to fit the needs of a particular instructor or group of students. Figure 2 shows how the various chapters are interdependent or independent, allowing progression through the chapters without loss of continuity.

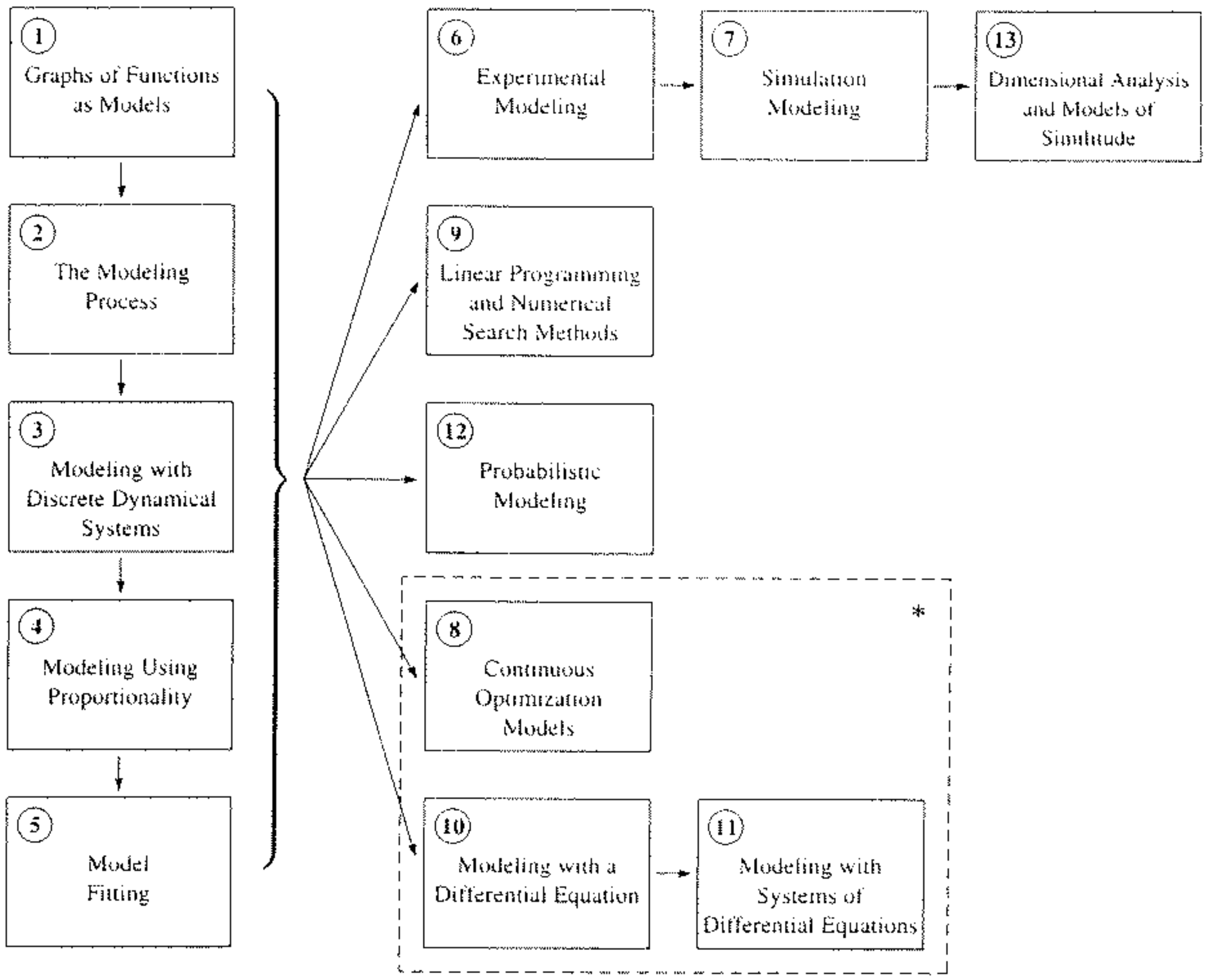


FIGURE 2 Chapter organization and progression

Student Projects

Student projects are an essential part of any modeling course. This text includes projects in creative and empirical model construction, model analysis, and model research. Thus we recommend a course consisting of a mixture of projects in all three facets of modeling. These projects are most instructive if they address scenarios that have no unique solution. Some projects should include real data that students are either given or can readily collect. A combination of individual and group projects can also be valuable. Individual projects are appropriate in those parts of the course in which the instructor wishes to emphasize the development of individual modeling skills. However, the inclusion of a group project early in the course gives students the exhilaration of a "brainstorming" session. A variety of projects is suggested in the text, such as constructing models for various scenarios, completing UMAP(3) modules, or researching model presented as an example in the text or class. It is valuable for each student to receive a mixture of projects requiring either model construction, model analysis, or model research for variety and confidence building throughout the course. Students might also choose to develop a model in a scenario of particular interest, or analyze a model presented in another course. We recommend six to eight short projects in a typical modeling course. Detailed suggestions on how student projects can be assigned and used are included in the Instructor's Manual that accompany this text.

In terms of the number of scenarios covered throughout the course, as well as the number of homework problems and projects assigned, we have found it better to pursue a few that are developed carefully and completely. Two or three good problems are about the maximum that an average student can handle in one week. We have provided many more problems and projects than can reasonably be assigned to allow for a wide selection covering many different application areas.

The Role of Computation

Although computing capability is not a requirement in using this book, computation does play an importanl role in several chapters, especially Chapters 3 and 9 where a spreadsheet is beneficial, and Chapter 7 where a means of generating random numbers is essential. We have found a combination of programmable calculators and microcomputers (or graphing calculators) to be advantageous throughout the course. Students who have programming experience can write computer code as part of a project, or software can be provided by the instructor as needed. We include some programs in the Instructor's Manual for this text. Typical applications for which students will find computers useful are in graphical displays of data, transforming data, least-squares curve fitting, divided difference tables and cubic splines, programming simulation models, linear programming and numerical search methods, and numerical solutions to differential equations. The use of computers has the added advantage of getting students to think early about numerical methods and strategies, and it provides insight into how "real-world" problems are tackled in business and industry. Students appreciate being provided with or developing software that can be taken with them after completion of the course.

Resource Materials

We have found material provided by the Consortium for Mathematics and Its Application (COMAP) to be outstanding and particularly well suited to the course we propose. COMAP was started from a National Science Foundation grant and has as its goal the production of instructional materials to introduce applications of mathematics into the undergraduate curriculum.

Individual modules for the undergraduate classroom, UMAP Modules, may be used in a variety of ways. First, they may be used as instructional material to support several lessons. ln this mode a student completes the self-study module by working through its exercises (the detailed solutions provided with the module can be conveniently removed before it is issued). Another option is to put together a block of instruction using one or more UMAP modules suggested in the projects sections of the text. The modules also provide excellent sources for "model research," because they cover a wide variety of applications of mathematics in many fields. In this mode, a student is given an appropriate module to research and is

(3) UMAP modules are developed and distributed through COMAP. Inc. 57 Bedford Street, Suite 210, Lexington, MA 02173.

asked to complete and report on the module. Finally, the modules are excellent resources for scenarios for which students can practice model construction. In this mode the instructor writes a scenario for a student project based on an applicalion addressed in a particular module and uses the module as background material, perhaps having the student complete the module at a later date.

Projects with an interdisciplinary perspeclive (ILAPs) are being developed by a consortium led by West Point and distributed by COMAP under a grant (from 1996 to 2000) from the National Science Foundation. The projects are designed by interdisciplinary teams of faculty and include both individual and group work. Information on the availability of interdisciplinary projects can be oblained by writing COMAP at the address given previously, calling COMAP at 1-800-772-6627, or electronically: order@comap.com

The Mathematical Contest in Modeling

The first Mathematical Contest in Modeling (MCNI) was held in 1985. Founded by Ben Fusaro, its purpose was to awaken interest in mathematical modeling. Each year two problems, one continuous and one discrete, are posed. On a designated weekend in February, teams may work from Friday morning until Monday afternoon and are free to consult any inanimate resource. Facully advisors prepare the teams tor the competition and ensure that the rules are complied with, but they are not consulted after the contest begins. Teams are encouraged to use any technology. Teams submit a complete solution with an abstract to the problem of their choice. Judges classify the papers into four categories: Outstanding, Meritorious, Honorable Mention, and Successful Participant. Outstanding teams are invited to present their papers at meetings of professional societies supporting the contest. Winning solutions are published in the Fall edition of the UMAP Journal. The Contest problems for the years 1985-1996 are presented in Appendix A. A special edition of the UMAP journal, MCM the First 10 Years, contains winning solutions and hints for the facully advisor, and is available from COMAP.

In 1996, 393 teams from 225 schools competed in the contest. The contest is sponsored by COMAP with funding support from the National Security Agency, the Society of Industrial and Applied Mathematics, the Institute for Operations Research and the Management Sciences, and the Mathematical Association of America. Additional information concerning the contest can be obtained by contacting COMAP.

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