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## Preface

The art of applying mathematics to real problems, be they in engineering, geophysics, industry, biology, or any other discipline, is one that is of enormous importance but which is also under relentless pressure: from computational scientists on the one hand and from mathematical analysts on the other. This book is about the middle ground between the two, and in it I hope to demonstrate that mathematical modeling is a subject of enormous potential, and one that has an essential role to play in many areas of modern applied science.

The book is hard, and intentionally so. There are many books on mathematical models that are aimed at a simpler expository level, but what I have endeavored to do here is to provide an insight into a way of thinking, which addresses problems more deeply. The only comparable book that I know of is that by Alan Tayler (1986). The difference between that and this is to some extent the scope of the problem areas and also the detail that I aim to go into. The earlier book by Lin and Segel (1974) is similar in ethos but much simpler.

It is possible to use this book as the basis for a course at graduate level, and indeed it embodies a course I have taught for the past several years at Oxford, in the M.S. program in Mathematical Modeling and Numerical Analysis. In that context, I use (for example) Part three as a rapid refresher/induction for basic models of partial differential equations, and then treat a selection of problems from Part four, two lectures per chapter. If you want to use this book as a course text, then you need to be a smart instructor -that is to say, the text is unforgiving, and some of the problems at the end of each chapter are at the level of research exercises. In the spirit of modeling, they may be more about finding an answer rather than finding the answer.

In writing this book around the course notes, I have added an introductory four chapters for completeness and reference, and I have also added four chapters at the end, which are more at the level of a research topic; indeed, each of them originated as an M.S. dissertation by former graduate students of mine.

Many people offered comments and advice on various chapters in the text, and I would like to thank them here for the efforts they have made in doing so: Alistair Fitt, Sam Howison, Jeff Dewynne, John Hinch, David Acheson, Hilary Ockendon, Malcolm Hood, John Willis, John Ockendon, Philip Maini, John Tyson, Lynn Van Coller, Donald Ludwig, Sean McElwain, Andrew Lacey, David Scott, Gary Parker, Joe Walder, Chris Huntingford, Chris Aldridge, Paul Emms, Richard Hindmarsh, Leslie Morland, Kolumban Hutter, Giri Kalamangalam, Chris Noon, John Holden, Guy Kember, and Steve Davis. I hope I haven't forgotten anybody. I also want to thank Ross Mackay, Bernard Hallet, Bill Krantz, and Claude Jaupart for providing copies of original photographs. Pat Black and John Holden were generous in providing photographs, which I was unfortunately unable to use. The image in exercise 16.6 was kindly provided by Michael Manga, who responded promptly and generously to my request. A morning photographic session in Rosie O'Grady's pub in Oxford showed that it is not easy to take pictures of Guinness !

I wish to thank also Don Drew, Richard Alley, Peter Howell, Felix Ng, and Taryn Malcolm for help in various ways; Brenda Willoughby for her usual flawless manuscript production; Lesley Cosier, who did the line drawings rapidly and efficiently; Alan Harvey and Amy Thomas at Cambridge University Press. A long time ago I went for a job interview at Cambridge University Press, and when I got the rejection letter, it said "We think it more likely that you will write books for us rather than publish them." Rather observant. Despite all my efforts, there will inevitably be errors, but I hope they are not too terrible.

This book is dedicated to the memory of Alan Tayler. He was my thesis supervisor, and in his relaxed way inspired me and a host of others to carry the torch of mathematical modeling into the murky depths of science and industry.

Oxford, March 1996