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Preface

Spurred on by past successes of mathetatical modelling in the physical sciences, more recent successes in the biological sclences and the advent of cheap and powerful computing, the use of mathematical models has grown rapidly and spread to most disciplines. There is every reason to believe that this trend will continue to accelerate in the foreseeable future. The very recent growth of the use of models for examining problems arising in industry is especially marked. Industrialists have realized that mathematical models, when combined with computation, represent a far cheaper alternative to other options (experimentation etc.) for understanding many industrial processes. Basically, modellers and computing come cheap! It is hoped that this book will provide useful material for the training of students in the skills of mathematical modelling in general, and in industrial mathematics in particular. It's also hoped that the book will expose students to the challenges and fascination of mathematical modelling as a profession.

The attitudes and skills required for modelling are different from, but not incompatible with, those required for pure mathematics, and even differ from those normally emphasized in traditional applied mathematics courses. The authors believe that such attitudes and skills should be developed early in a student's background, and the only obvious way to do this is to expose students to real modelling problems. More specifically, students need to develop the ability to work with non-mathematical material presented in succinct form and to learn how to formulate models, determine solutions which are useful in context, and interpret the results. The need for such early modelling training has been generally recognized but, given the limited mathematical and contextual background of such students, and their limited exposure to the practical use of mathematics, it's not easy to design such courses. We've attempted to do this by carefully selecting the material (especially the models) and by using an algebraic package to provide technical assistance.

The book requires as mathematical background no more than second year calculus level (elementary differential equations, vector calculus etc.), together with the associated linear algebra, and assumes a small non-mathematical knowledge base. The non-mathematical background require is briefly discussed in the relevant chapters. The models examined relate largely to industrial and (to a lesser extent) scientific questions that the authors have encountered recently, and which are seen to be of current and future practical interest. Particular situations are presented (often as presented to the authors by the customer) and what follows is a discussion of the alternative approaches which may be adopted, followed by an analysis of the relevant models and an appraisal of the practical usefulness of the results obtained. Techniques are introduced when needed, and the advantages and disadvantages of such techniques are indicated in context. Errors in approach are also part of the learning experience and are presented. In a real sense an apprenticeship approach is taken.

Additionally the material has been selected so as to:

-Illustrate the use of mathematical techniques which are efficient practical, and frequently used.

-Reinforce significant ideas by meeting them in more than one context.

The cohesiveness necessary to do this places restrictions on content. For this reason the large scale discrete optimization problems and the statistical problems which often arise in industrial contexts were excluded. Other computer intensive models were excluded for the same reason.

-Present as much of the material normally regarded as essential for continuum modelling as possible. It is in the manner of presentation of this (and other) material that this book differs from traditional presentations.

-Draw the student's attention to the features that make different systems behave in similar ways. An awareness of such features is essential for modelling; this is why dynamics, diffusion and wave propagation are fundamental topics. Of course it's this commonality feature that enables modellers to contribute significantly over such a broad front.

One major difficulty encountered by students is that of handling straightforward but intricate and tedious technical calculations. This is a difficulty experienced by all mathematicians; however, with experience, one learns to avoid getting bogged down in the technical details. For the apprentice and professional alike symbol manipulation packages are a Godsend. At the press of a button Fourier coefficients to any order are calculated, differential equations solved, plots drawn etc. This enables one to direct one's mind to major issues such as the appropriateness of the solution procedure or the relevance of the calculations in the modelling context. It should be pointed out that such packages are not just a convenient tool for performing calculations - the methodology of mathematics is rapidly changing as a result of their introduction. Apart from the fact that such packages must be regarded as essential for present day mathematics usage, the use of Maple, just one of the competitive packages, in this book has enabled a realistic working approach to modelling to be adopted; students are stepped through real calculations. Thus using Maple has enabled the authors to present techniques that are commonly used in practice (rather than those restricted by class-time constraints) and to examine realistic models. Also, students are expected to carry out (close to) real size investigations, rather than the artificially simplified (often superficially related) calculations necessitated by time constraint in the pre-algebraic package era. It would be helpful if students had some familiarity with Maple but it's not essential; students are introduced slowly to the commands in the first few chapters and more commands are introduced when needed. The authors have in practice found it useful to present one lecture illustrating the capabilities of the package and refer students to notes or any of the many good books on Maple (eg. Ellis, Johnson, Lodi and Schwalbe (1992), or Heck (1993)). An up to date version of the Maple code used in the book will be maintained at the web site http://maths.uwa.edu.au/~fowkes/homepage.html, where worked solutions to exercises will also be available. Alternative algebraic packages can be used in association with the text, and Mathematica code for the book will also be maintained at the above site.

An unusual exercise format is adopted to facilitate the handling of these real problems. The context of the problem is explained and then a series of directives is given to assist the student's own investigations. Hints are provided to enable the student to discover a sensible approach, and the student is expected to interpret results in context, and is encouraged to experiment and suggest extension work. Initially this approach is somewhat daunting for students (for starters many of the exercises are rather longer than usual and more open ended); however, it has been found that confidence quickly develops. The examples form an essential component of the text. They either elaborate the text or examine analogous situations arising in often entirely different contexts. The connecting features are thus emphasized, and also it's hoped that students will develop the confidence to handle situations outside their normal experience and build up a useful knowledge base.

In order to facilitate the apprenticeship approach an informal conversational style is used. As we all know, mathematicians don't think the way they write. They write like lawyers - and no one can think like a lawyer writes! In writing the way they think the author hope that the intuitional model that enable them to see through situations might also prove to be of value to students. At the very least it's hoped that the approach will encourage students to develop their own intuitive way of looking at situations.

The book is an expanded version of a course given mainly by the authors and Malcolm Hood to second year university students at the University of Western Australia on mathematical modelling. Malcolm's imprint on a number of the exercises will be recognized by students. The course evolved from a standard course on applied mathematics into a course based on various scientific investigations we have personally been involved in (of local and global origin) and later, as our interests spread, on industrial problems. Our interest in industrial problems was stimulated especially by the work of the Oxford Industrial Mathematics group, and the annual Australian Mathematics-in-Industry meetings run until this year by CSIRO.

A great deal of this book was completed during the last year of John's life when he was struggling with cancer; a very difficult period for him and his family. In spite of very limited energy resources John insisted on continuing because of his belief in the importance of such a work. The example of his courage is at least as valuable as the work contained in this book. The work would not have been possible if it wasn't for Jocelyn's (John's wife) extraordinary support; for this we're all grateful.

Many friends and colleagues have offered encouragement, provided welcome advice, corrected errors etc. I'd especially like to acknowledge the encouragement and advice from Bill Pritchard and Glyn Davies, and to thank Jenny Hopwood for critically commenting on the book material. Peter Chapman also offered useful advice on various chapters of the book, and Grant Keady offered Maple advice. I'm also grateful to my good friends Roman Bogoyev and Con Savas who were most generous with their time and help.

Working with John during the last year of his life was distressing, and putting the material together since his death has been most difficult. Without the help of dear friends (especially Paula) I could not have managed. All I can say is thanks!

Nev Fowkes John Mahony

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