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Series Preface

This series is designed particularly, but not exclusively, for students reading degree programmes based on semester-long modules. Each text will cover the essential core of an area of mathematics and lay the foundation for further study in that area. Some texts may include more material than can be comfortably covered in a single module, the intention there being that the topics to be studied can be selected to meet the needs of the student. Historical contexts, real life situations, and linkages with other areas of mathematics and more advanced topics are included. Traditional worked examples and exercises are augmented by more open-ended exercises and tutorial problems suitable for group work or self-study. Where appropriate, the use of computer packages is encouraged. The first level texts assume only the A-level core curriculum.

Professor Chris D. Collinson

Dr Johnston Anderson

Mr Peter Holmes

Preface

'I have yet to see any problem, however complicated, which, when looked at in the right way, did not become still more complicated' , Paul Anderson

This book is about the use of mathematics to solve problems in the real world. Traditionally the discipline in which the use of mathematics is studied has been called Applied Mathematics, but this term has often been associated with the application of mathematics to science and engineering. Often, in school, Applied Mathematics is associated with Mechanics, but mathematics occurs in many other subjects, for example in economics, biology, linguistics, transport as well as in industry, commerce and government.

Applying mathematics to such a wide range of subjects requires not only good mathematical problem solving skills but the ability of the mathematician to start with a problem in non-mathematical form and to give the results of any mathematical analysis in non-mathematical form. In between these starting points are the important skills of mathematical modelling. The process of mathematical modelling consists of three main stages; we take a problem set in the real world and first formulate it as a mathematical problem; this together with any assumptions made is the mathematical model. The mathematical problem is then solved and finally the solution is translated back into the original context so that the results produced by the model can be interpreted and used to help solve the real problem.

To become skillful at mathematical modelling requires much hard work through experience gained at problem solving. This book will help you to develop mathematical modelling skills by solving problems; it is a "doing book" not a "watching book" in the sense that it is important to have a go at some of the problems posed -you will not become a good modeller by watching others.

The aim of this book is to develop your mathematical modelling skills in three different ways:

- by solving simple modelling problems;
- by developing mathematical models in one application area, population growth;
- by reading and giving a critical appraisal of published articles.

The book has four chapters. Chapter 1 introduces the mathematical modelling process through a wide range of real problem situations and answers the question 'what is mathematical modelling?' Chapter 2 takes one particular application of mathematics to population growth through which the mathematical modelling process is demonstrated. Chapter 3 contains reprints of six articles from published books showing mathematical modelling in action. The author of each article has developed one or more mathematical models to describe a particular situation and has used the models to answer questions about the situation. The aim of this chapter is to encourage you to see the modelling process at work when you read mathematics. Chapter 4 contains activities to develop your mathematics modelling skills further. Section 4.1 has more general problems to solve whereas Sections 4.2 and 4.3 introduce two alternative approaches to formulating models; simulation modelling and dimensional analysis modelling.

Study Guide

The book is written to support a first course in mathematical modelling. It is likely that not all the material will be used by every student on such a course. Our experience has shown that it is important in any modelling course to have a range of application areas for students from different backgrounds and interests. On our modelling courses we are often seeing students majoring in the sciences, social sciences, humanities as well as mathematics.

We would recommend that students study Chapters 1 and 2 fully as the material therein is designed to develop modelling skills as well as seeing the power of mathematics in modelling population growth. Subsequently, we would recommend that each student studies two of the articles from Chapter 3 and one section from Chapter 4; the choice depending on their main subject area. The modelling problems in section 4.1 can be used to consolidate the modelling skills developed in the earlier chapters; section 4.2 is probably suitable for students with a statistics background whereas Section 4.3 is for those with a mechanics or science background. We hope that Chapter 4 will provide a book for the diverse interest of students following modular type courses.

This book is the result of many years of teaching mathematical modelling to undergraduates and teachers on full-time and part-time courses at the Universities of Plymouth, Ulster and the Open University. Hence there are many colleagues in school, further and higher education with whom discussions over the past fifteen years have helped formulate our thoughts and courses in modelling. We would like to thank them and other people who has influenced the production of this book. Not wishing to upset those whose names we forget we have decided not to name any of these colleagues. But you know who you are!

So thanks!!

We are grateful to the following publishers for permission to reproduce material from the following texts:

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M371 Block IV Unit 1;

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We thank Mrs Sharon Ward for turning our handwriting and our poor typing into the excellent CRC for this book. We appreciate her patience and advice in the many changes to the page design and layout. We also thank Michael Broughton for preparing the artwork.

To the reader, we wish good modelling!

John Berry

Ken Houston