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PREFACE

Whilst there are a number of recent texts in mathematical modelling of the 'case study' variety, these are generally of most use as source material for the teacher. This Guide to Mathematical Modelling, however, is intended to be read by students; so the topics treated and the order of contents have been chosen with this in mind. We have tried to address the problem of how mathematical modelling is done as well as what a mathematical model is, and so have avoided presenting just a long catalogue of completed modelling examples.

The book is essentially a first course; so the amount of prerequisite mathematics and statistics is quite modest. It is chiefly aimed at the first-year level in an undergraduate degree course in mathematical sciences, but the treatment is such that the book could be used in the second year of a school sixth form. The contents have formed the basis of the first-year modelling course for students studying for B.Sc. in Mathematics, Statistics and Computing at the University of Greenwich (formerly Thames Polytechnic) and have proved a successful component in this course. We also gratefully acknowledge the influence of the pioneering work of many colleagues from the Open University and the former Polytechnics in the area of teaching mathematical modelling. The book stops short of investigating large-scale simulation models requiring software packages, but it lays valuable groundwork for subsequent study of such models.

At the outset, it is important to explain not only what modelling is, but also why it is worth doing. It is not merely a means of making the usual first-year curriculum in mathematics and statistics more lively and applicable. To accept that is to miss the point. The objective is to provide an approach to formulating and tackling problems in terms of mathematics and statistics. Eventually, when entering employment where real problems have to be dealt with, mathematicians will require additional skills to those fostered by study of conventional topics on the curriculum. The study of modelling promotes the development of these extra skills.

The book is divided into 10 chapters. Although it is not necessary to read the book strictly in chapter order, this may be preferred since there is some progression in difficulty as the subject is developed. It is vital, however, that readers try their hand at solving many of the problems posed, since modelling skills can only be learned by active participation.

Having set the scene in the opening chapter, some simple modelling problems are presented in chapter 2. These come from a variety of backgrounds, and readers should try some of the examples themselves from the problem descriptions provided. Mathematical modelling is by its nature difficult to structure, but it is useful to lay down general guidelines within which to operate when faced with new situations. To this end a general methodology is described in chapter 3.

The succeeding three chapters are particularly important for the beginner. Here the essential skills for successful modelling are developed. These are as follows.

1 Identifying the problem variables.

2 Constructing appropriate relations between these variables.

3 Taking measurements and judging the size of quantities.

4 Collecting data and deciding how to use them.

5 Estimating the values of parameters within the model that cannot be measured or calculated from data.

The backbone of the text comes in chapters 7 and 8. Chapter 7 deals with approaches to problems involving random features which demand some statistical analysis. Chapter 8 covers modelling situations which give rise to differential equations, such as are often encountered in physics and engineering.

Communication is vital for successful implementation of a mathematical model. It is necessary to explain ideas behind a model to other people, some of whom may not necessarily hold the same opinion as the modeller. It is also necessary to advise on the use of a model, often to non-specialists who need only to understand the essential points. Further, both at college and later in employment, it is often necessary to present findings verbally to a small group. These communication skills do not always come naturally; so, in chapter 9, advice is given on these matters.

Finally, in chapter 10, more demanding modelling assignments are presented. Some of the models are fully developed but others are left for the reader to process.

The content of this book complements other material usually studied in a mathematics degree course, and there is plenty of scope for further work in modelling as experience in mathematics and statistics is increased. Solving real problems by mathematical modelling is a challenging task, but it is also highly rewarding. If by working through the book readers gain confidence to take up this challenge, then the authors will be satisfied that the effort of writing the book has been worthwhile.

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