TAMPERE UNIVERSITY OF TECHNOLOGY

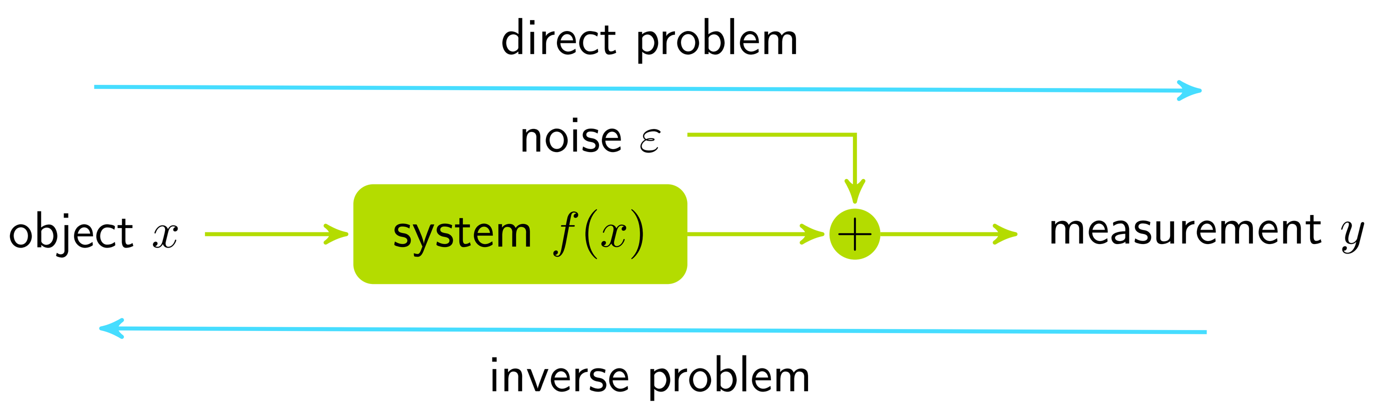
INVERSE PROBLEMS

Inverse problems research group

Department of Mathematics

What are inverse problems?

Inverse problems are *ill-posed* mathematical problems that are non-trivial to solve. Consider the following example of a measuring system.



The direct problem simply solves what measurements is received from a given input x. The inverse is naturally the opposite of that: solve what input caused the measurement. If

$$y = f(x) + \varepsilon, \tag{1}$$

then what is x? One could think it is simply

$$x = f^{-1}(y - \varepsilon), \tag{2}$$

however there are several problems.

Existence The inverse system $f^{-1}(\cdot)$ might not exist, and furthermore there is no guarantee that an object x giving the measurement y even exists.

Uniqueness An infinite number of different inputs $x_1, x_2, \ldots$ can cause the same output. How to choose the *correct* one?

Stability Even if the original system is bounded, the inverse system with a noisy input might not be.

Inverse problems at TUT

From biology to cosmology, many of the topics we study are about surface reconstruction as an inverse problem, in which field we are an internationally leading group.

In addition to deriving fundamental mathematical results on various types of inverse problems, we develop reconstruction procedures that converge efficiently and can handle large data sets. Our main application fields are space research, forest and environmental sciences, and remote sensing.

Our projects are part of the research programme of the Finnish Centre of Excellence in Inverse Problems Research and they are funded by the Academy of Finland and Tampere University of Technology. The centre of excellence status has been granted for the periods 2006–2011 and 2012–2017.

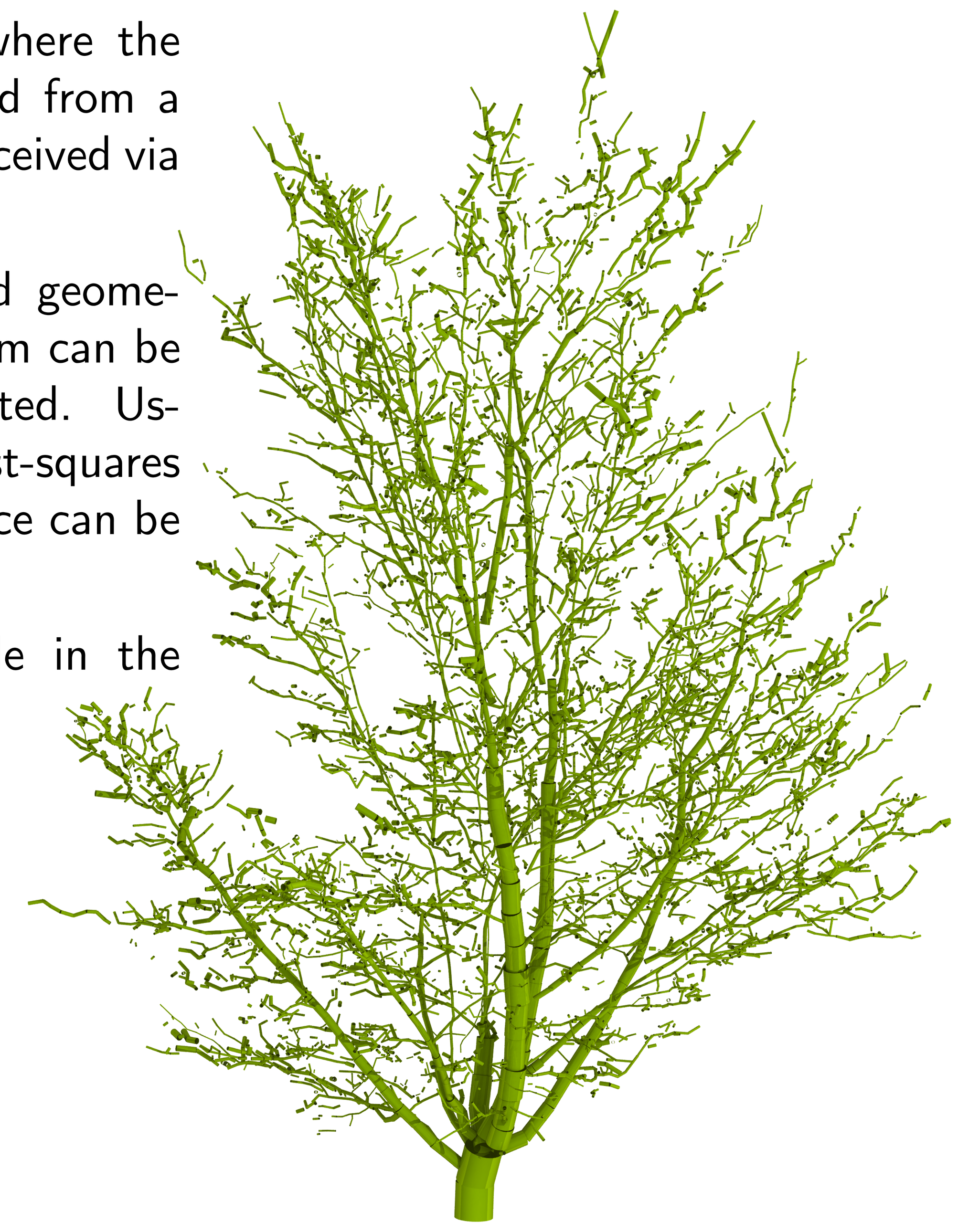
Finnish Centre of Excellence
in Inverse Problems Research

Multidimensional reconstruction

Surface reconstruction is a crucial inverse problem also in dimensions larger than three. Regular motion in Hamiltonian 3D systems is confined to 3-tori in six-dimensional phase space. The dynamics of a near-integrable system, such as our galaxy, can be described by perturbations of a foliated set of tori in 6D that defines the mass distribution and potential field of the system.

Thus, the reconstruction of these surfaces from observations yields a well-defined model of the system, and especially the dark matter contained in it. We develop practical generally applicable procedures for torus construction, to be used in, e.g., the analysis of large-scale sky surveys.

Complex surface structures



An environmental application, where the surface of a tree is reconstructed from a set of three-dimensional points received via terrestrial laserscanning.

By using the local topology and geometry of the measurements, the stem can be found and the branches segmented. Using further subdivision and least-squares fitting, the local volme and surface can be approximated as a cylinder.

The following data are available in the model:

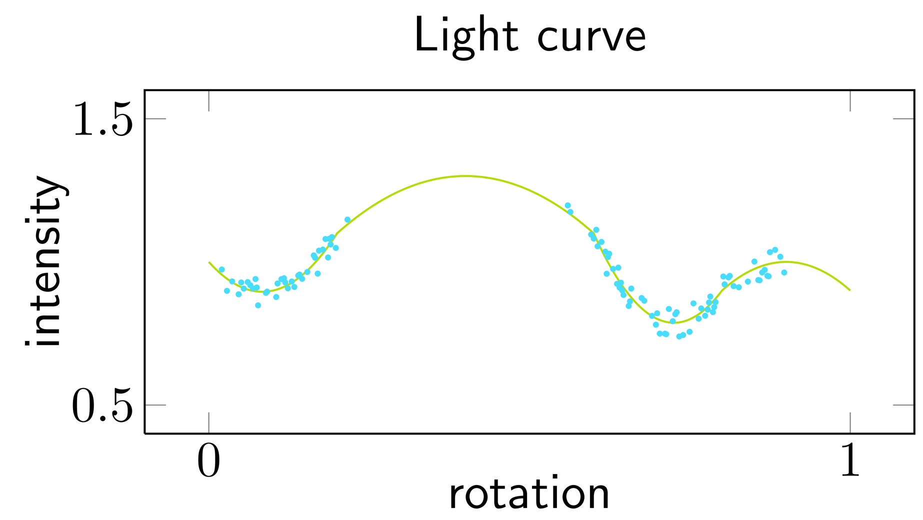
- volume distribution
- stem taper profile
- stem lean and sweep
- branching angles
- branching topology.

Generalized projections

In close collaboration with several international teams, our team develops procedures for asteroid modelling using various available data sources. Most asteroid models in existence (several hundreds by now) have been constructed with our techniques.

Data sources include:

- flyby data from space missions
- interferometric projections
- thermal infrared radiation
- adaptive optics data
- radar measurements
- stellar occultations
- photometry

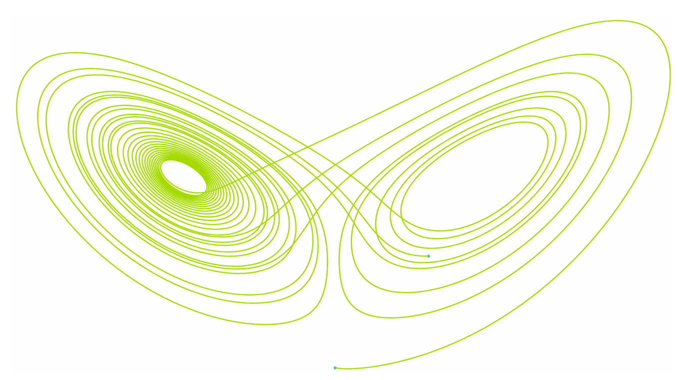


Rendered 3D model of asteroid Hygiea

Courses

MAT-62006 Inverse Problems An introductory course to inverse problems. The course concentrates on how to recognize an inverse problem and how to solve it in practice, even when the data are noisy and the number of unknowns is large. Includes a strongly recommended, application oriented project work. Course is part of the activities of the Finnish Centre of Excellence in Inverse Problems Research.

MAT-61506 Dynamical systems and chaos Introduction to nonlinear dynamical systems and chaotic dynamics. Examples of such systems include biology, circuitry, meteorology, Hamiltonian systems and mechanics, discrete systems, etc. The course includes a project work on computational aspects.



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