Alternating group sparsity for image restoration

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Recently, collaborative image filtering based on group-based sparse representation has gained a popularity in image restoration. BM3D frame [1], one of the first example of such a representation, utilizes both local sparsity of small size image patches and group-sparsity of collections of self-similar image patches. As a sparsifying transforms in the spatial and similarity domains, fixed transforms (e.g., DCT or wavelets) or data-adaptive transforms (obtained by SVD or PCA) [4] can be used. Modern image restoration methods utilize l0 and l1 minimization frameworks applying convex optimization algorithms, e.g. iterative shinkage-thresholding, split Bregman (SB), etc. In [5], group sparse representation (essentially, BM3D frame with SVD as a sparsifying transform [4]) is used in the framework of SB, whereas in [1] BM3D frames with fixed sparsifying transform have been used.

This paper generalizes both methods of [5] and [1]. Instead of applying BM3D frames with a single fixed transform or with SVD, we propose to use multiple BM3D frames in iterations in alternating fasion. Specifically, in the derivation and the algorithm implementation, we arrive to the following two types of filters: BM3D with the hard-thresholding in SVD domain (SVD computed for matrices whose columns are similar patches represented as the vectors) for the basic design, interleaved with the empirical $1D$ Wiener BM3D where the intermediate estimate exploited as the reference image. Here $1D$ means that the Wiener filtering is applied only in the similarity direction, i.e. to the rows of $2D$ group representations. These filters are applied to the high-resolution variables (estimates and residuals of the estimates) and work alternatively. Thus, the grouping in iterations is produced more than ones, it is applied to different variables and the priors used are different. The algorithm can be described as an iterative shrinkage based on adaptive self-learning dictionaries interleaved with an empirical one-dimensional Wiener filtering in the similarity domain.

Overall, this work is in line with the current trends in image processing, that the accurate formulation of the prior can be omitted in favor of a good denoising algorithm embedded in the iterations (see, e.g., [2]). We prove that indeed the used priors results in the efficient filters and, even more, alternative application of these filters leads to the extraordinary good results.

A linear ill-posed inverse problem, typical for image restoration, in particular, for image denoising, deblurring and super-resolution, is considered here

$$y = Hx + \varepsilon, \tag{1}$$

where $y \in \mathbb{R}^m$, $x \in \mathbb{R}^n$, $m \leq n$, H is a known linear operator and $\varepsilon \in \mathbb{R}^m$ is an additive Gaussian noise.

The problem is to resolve (1) with respect to x provided some prior information on x. In terms of super-resolution, the operations in $\mathbb{R}^m$ and $\mathbb{R}^n$ can be treated as operations with low-resolution and high-resolution images, respectively. Usually the iterative algorithms to estimate x from (1) include both upsampling and downsampling operations and some prior information on these variables.

Let us give some notes concerning the algorithms as they are specified in particular for deblurring and upsampling. For image deblurring, with $m = n$ and Gaussian noise, we start from the quadratic criterion $J_0 = ||Hx-y||_2^2+\gamma_0||x-\hat{x}||_2^2/2$, where $\hat{x}$ is a *sparse* estimate of x. For $m = n$, the solution of $\min_x J_0$ is the regularized inverse $x(\hat{x}) = \dfrac{H^T y}{H^T H + \gamma_o I} + \gamma_o \hat{x}$, defining the link between x and $\hat{x}$. The priors on x are applied to the auxiliary variable $\hat{x}$ in the form of developed mentioned above alternating BM3D and empirical $1D$ Wiener BM3D filters. Updates of $\hat{x}$ and x define the iterative algorithm.

For upsampling, $m \leq n$, we us start from the gradient procedure for $J_0 = ||Hx - y||_2^2$ in the form $x^{t+1} = x^t - \alpha H^T[Hx^t - y]$. Assume that $\uparrow$ denotes an *approximate upsampling operation* including the deblurring of downsampling, i.e. $\uparrow \simeq H^T$, respectively $\downarrow \simeq H$, then this iterations take the form

$$x^{t+1} = x^t - \alpha[x^t \downarrow - y] \uparrow . \tag{2}$$

The developed alternating BM3D and empirical $1D$ Wiener BM3D filters are applied to x^t implimenting priors on x and define in this way the proposed iterative upsampling algorithm.

Multiple experiments on image deblurring and upsampling confirm an advantage in performance of the proposed method compared to the best results in the field. For deblurrring, we have compared our method with the state-of-the-art GSR and IDD-BM3D on several standard datasets. Overall, proposed aproach outperforms them in subjective and objective quality (such as PSNR, PSNR-HVS-M, FSIM, etc.) terms. Two examples of image deblurring are given below. First, the test image 'Barbara' of size 256x256, was blurred with the Gaussian blur, kernel 25x25, std = 1.6, and corrupted by an additive white Gaussian noise with noise variance equal to 2. The PSNR results of deblurring are the following: by GSR - 28.6 dB,

Fig. 1. Examples of image deblurring (Image 'Barbara' 256x256, From left to right: blurred image, Gaussian blur (kernel 25x25, std = 1.6); noise added variance 2; deblurring by GSR , PSNR = 28.6 dB, deblurring by our method, PSNR = 28.8 dB)

Fig. 2. Examples of image deblurring (Image 'Barbara' 256x256, From left to right: blurred image, Gaussian blur (kernel 25x25, std = 1.6); noise added variance 2; deblurring by GSR , PSNR = 28.6 dB, deblurring by our method, PSNR = 28.8 dB)

by IDD-BM3D - 27.2 dB, and by our method - 28.8 dB. Fig. 1 shows the blurred image 'Barbara', and the results of deblurring by GSR and by our method. Second example of image deblurring is shown for the image 'cameraman' of size 256x256, blurred by the uniform blur kernel of size 9x9, and corrupted by an additive white Gaussian noise with variance 2. The PSNR results of deblurring are: by GSR - 30.8 dB, by IDD-BM3D - 31.2 dB, and by our method - 31.3 dB. Fig. 2 shows the blurred image and the results of deblurring by IDD-BM3D and by the proposed method.

For super-resolution, the proposed approach was compared with the recently proposed methods [3], [6] on standard image databases for different up-sampling factors. Our method achieves superior results. It works well not only on the Hreal world natural images, but also on images with substantial urban structure. Table 1 shows PSNR comparison on two image datasets for upscaling ratios 3 and 4.

TABLE I
PSNR COMPARISON ON TWO DATASETS AMONG DIFFERENT METHODS OF IMAGE SUPER-RESOLUTION FOR RESAMPLING RATIOS X3 AND X4 (BICUBIC, A+, CSCN-MV, OUR)

Set5, ratio	Bicubic	A+	CSCN-MV	our
x3	30.39	32.59	33.34	33.71
x4	28.42	30.29	32.80	33.54
Set14, ratio				
x3	27.54	29.13	29.57	29.79
x4	26.00	27.33	27.81	28.06

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